習題解答

第一章

1.

(a) $R_T = \dfrac{P}{4\pi r^2} = \sigma T^4$

$$T^4 = \frac{P}{4\pi r^2}\frac{1}{\sigma} = \frac{(3.85\times 10^{26}\text{w})}{4\pi(6.96\times 10^8\text{m})^2\times\left(5.67\times 10^{-8}\dfrac{\text{w}}{\text{m}^2\text{-K}^4}\right)}$$

$$= 11.1545\times 10^{16}\text{K}^4$$

$$\therefore T = 5778\text{K}$$

(b) $\lambda_{\max} = \dfrac{2.898\times 10^{-3}\text{m-K}}{T(\text{K})} = \dfrac{2.898\times 10^{-3}}{5778}\text{m} \simeq 5015.5\text{Å}$

2.

(a) 總輻射能量為

$$P = 4\pi r^2 R_T = (4\pi r^2)(\sigma T^4)$$

$$= 4\pi(0.7\times 10^9\ \text{m})^2\left(5.67\times 10^{-8}\frac{\text{w}}{\text{m}^2\text{-K}^4}\right)(5700\text{K})^4$$

$$= 3.68\times 10^{26}\ \text{W}$$

而 $P = \dfrac{d}{dt}(mc^2) = c^2\dfrac{dm}{dt}$

$$\frac{dm}{dt} = \frac{P}{c^2} = 4.096\times 10^9\ \text{kg/s}$$

(b) $\Delta M = \left(\dfrac{dm}{dt}\right)\times 1\text{year} = 1.292\times 10^{17}\ \text{kg}$

$$\frac{\Delta M}{M} = \frac{1.292\times 10^{17}}{2\times 10^{30}} = 6.5\times 10^{-14}$$

3.

(a) $\lambda_{\max} T = 2.898\times 10^{-3}\ \text{m-K}$

$$\lambda_{\max} = \frac{2.898\times 10^{-3}\ \text{m-K}}{T} = \frac{2.898\times 10^{-3}\ \text{m-K}}{309\text{K}}$$

$$=9.38\times10^{-3}\,\mathrm{m}$$

(b) $A=2(3\times0.3)+2(3\times0.2)+2(0.2\times0.2)=3.12\mathrm{m}^2$

$$P=AR_T=A\,(\sigma T^4)=(3.12\mathrm{m}^2)\left(5.67\times10^{-8}\frac{\mathrm{w}}{\mathrm{m}^2\text{-}\mathrm{K}^4}\right)(309\mathrm{K})^4$$

$$=1633.55\mathrm{w}$$

4.

(a) $\dfrac{R_1}{R_2}=\dfrac{\sigma T_1^4}{\sigma T_2^4}=\left(\dfrac{T_1}{T_2}\right)^4=\left(\dfrac{1000}{2000}\right)^4=\left(\dfrac{1}{2}\right)^4=\dfrac{1}{16}$

(b) $(\lambda_{\max})_1\,T_1=(\lambda_{\max})_2\,T_2$

$$\therefore\frac{(\lambda_{\max})_1}{(\lambda_{\max})_2}=\frac{T_2}{T_1}=\frac{2000}{1000}=2$$

$$\frac{(v_{\max})_1}{(v_{\max})_2}=\frac{\left(\dfrac{c}{\lambda_{\max}}\right)_1}{\left(\dfrac{c}{\lambda_{\max}}\right)_2}=\frac{(\lambda_{\max})_2}{(\lambda_{\max})_1}=\frac{1}{2}$$

5.

$\lambda_{\max}\,T=\mathrm{cons}\,t\qquad R_T=\sigma T^4$

$$\therefore R_T=\sigma\left(\frac{\mathrm{cons}\,t}{\lambda_{\max}}\right)^4=\frac{\sigma}{\lambda_{\max}^4}(\mathrm{cons}\,t)$$

因此

$$2R_T=2\left(\frac{\sigma}{\lambda_{\max}^4}\right)\mathrm{cons}\,t=2\left(\frac{\sigma}{\lambda'^{\,4}_{\max}}\right)\mathrm{cons}\,t$$

$$\lambda'_{\max}=\left(\frac{1}{2}\lambda_{\max}^4\right)^{1/4}=\left(\frac{1}{2}\right)^{1/4}\lambda_{\max}=0.84\lambda_{\max}$$

$$=(0.84)(6500\text{Å})$$

$$=5460\text{Å}$$

6.

$\lambda_1 = 2000\text{Å} = 200\text{nm}$，$\lambda_2 = 4000\text{Å} = 400\text{nm}$

而 $\rho_T(\lambda) = \dfrac{8\pi hc}{\lambda^5} \dfrac{1}{e^{hc/\lambda kT} - 1}$

$$\therefore \frac{1}{\lambda_1^5} \frac{1}{e^{hc/\lambda_1 kT} - 1} = (3.82) \frac{1}{\lambda_2^5} \frac{1}{e^{hc/\lambda_2 kT} - 1}$$

$$\frac{e^{hc/\lambda_2 kT} - 1}{e^{hc/\lambda_1 kT} - 1} = (3.82)\left(\frac{\lambda_1}{\lambda_2}\right)^5 = (3.82)\left(\frac{1}{2}\right)^5 = 0.1194$$

$$\frac{hc}{\lambda_1 k} = \frac{(6.624 \times 10^{-34}\ \text{J-s})(3 \times 10^8\ \text{m/s})}{(2 \times 10^{-7}\ \text{m})(1.38 \times 10^{-23}\ \text{J/K})} = 71974\text{K}$$

$$\frac{hc}{\lambda_2 k} = \frac{(6.625 \times 10^{-34}\ \text{J-s})(3 \times 10^8\ \text{m/s})}{(4 \times 10^{-7}\ \text{m})(1.38 \times 10^{-23}\ \text{J/K})} = 35987\text{K}$$

$$\therefore \frac{e^{35987/T} - 1}{e^{71974/T} - 1} = 0.1194$$

令 $x = e^{35987/T} \quad \Rightarrow \quad x^2 = e^{71974/T}$

$$\therefore \frac{x - 1}{x^2 - 1} = 0.1194 \quad \Rightarrow \quad x = 7.375$$

$\therefore e^{35987/T} = 7.375$

$\ln(e^{35987/T}) = \ln(7.375)$

$$\therefore T = \frac{35987}{\ln(7.375)} = 18{,}000\text{K}$$

7.

$$v = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{9.8\text{m/s}^2}{1\text{m}}} = 0.5/\text{s}$$

$E = mgh = (0.1\text{kg} \times 9.8\text{m/s}^2) \times (3 \times 10^{-2}\ \text{m})$

$= 29.4 \times 10^{-3}$ Joule

$hv = (6.63 \times 10^{-34}\ \text{J-s})(0.5/\text{s}) = 3.32 \times 10^{-34}\ \text{J}$

$$\therefore n = \frac{E}{hv} = \frac{29.4 \times 10^{-3}\ \text{Joule}}{3.32 \times 10^{-34}\text{J}} = 8.855 \times 10^{31}$$

$= 88.55 \times 10^{30}$

$$\frac{\Delta E}{E}=\frac{hv}{E}=\frac{3.32\times10^{-34}\,\text{J}}{29.4\times10^{-3}\,\text{J}}=0.113\times10^{-31}$$

$$=11.3\times10^{-33}\quad \text{不能測得出}$$

第二章

1.

(a) $E_{\text{Na}}=hv=\dfrac{hc}{\lambda}=\dfrac{12.408\times10^{-7}\,\text{eV-m}}{5890\times10^{-10}\,\text{m}}=2.1\text{eV}$

小於 2.3eV，不能有光電效應。

(b) $E_{\text{Na}}=hv=\dfrac{hc}{\lambda_{cut}}=2.3\text{eV}$

$$\lambda_{cut}=\frac{12.408\times10^{-7}\,\text{eV-m}}{2.3\,\text{eV}}=5398\text{Å}$$

2.

截止電壓為 0.71V，則

$$W=eV_0=0.71\text{eV}=hv-W_0=\frac{hc}{\lambda}-W_0$$

$$W_0=\frac{hc}{\lambda}-0.71\text{eV}=\frac{12.408\times10^{-7}\,\text{eV-m}}{4910\times10^{-10}\,\text{m}}-0.71\text{eV}$$

$$=1.817\text{eV}$$

今截止電壓為 1.43V，則

$$1.43\text{eV}=\frac{hc}{\lambda}-W_0$$

$$\frac{hc}{\lambda}=1.43\text{eV}+1.817\text{eV}=3.247\text{eV}$$

$$\lambda=\frac{hc}{3.247\,\text{eV}}=\frac{12.408\times10^{-7}\,\text{eV-m}}{3.247\,\text{eV}}=3821\text{Å}$$

3.

截止電壓

$$K_{max}=eV_0=h\nu-W_0=\frac{hc}{\lambda}-W_0$$

$$hc=\lambda(eV_0)+\lambda W_0$$

(i) $\lambda=3000\text{Å}$，$V_0=1.85\text{V}$

$$hc=(3000\text{Å})(1.85\text{eV})+(3000\text{Å})W_0$$

(ii) $\lambda=4000\text{Å}$，$V_0=0.82\text{V}$

$$hc=(4000\text{Å})(0.82\text{eV})+(4000\text{Å})W_0$$

$\therefore W_0=2.27\text{eV}$——鈉的功函數

(a) $hc=\lambda(eV)+\lambda W_0=(3000\text{Å})(1.85\text{eV})+(3000\text{Å})(2.27\text{eV})$

$=12.36\times10^{-7}\text{ m-eV}$

$$\therefore h=\frac{12.36\times10^{-7}\text{ m-eV}}{(3\times10^{8}\text{ m/s})}$$

$=4.12\times10^{-15}\text{ eV-s}=(4.12\times10^{-15}\text{ eV-s})(1.602\times10^{-19}\text{ J/eV})$

$=6.60024\times10^{-34}\text{ J-s}$

(b) $W_0=2.27\text{eV}$

(c) 截止波長

$$W_0=h\nu_T=\frac{hc}{\lambda_T}$$

$$\therefore\lambda_T=\frac{hc}{W_0}=\frac{12.408\times10^{-7}\text{ eV-m}}{2.27\text{eV}}=5466\text{Å}$$

4.

(a) 光電子在磁場中的運動

$m\dfrac{\text{v}^2}{r}=eB\text{v}$，則 $p=m\text{v}=erB$

電子能量

$$E^2=p^2c^2+(m_0c^2)^2=(erBc)^2+(m_0c^2)^2$$

$$erBc=e(1.88\times10^{-4}\text{ tesla-m})(3\times10^{8}\text{ m/s})$$

$$=5.64\times10^{4}\,\mathrm{eV}=0.0564\mathrm{MeV}$$

$$E^2=(0.0564\mathrm{MeV})^2+(0.511\mathrm{MeV})^2=26.4302\times10^{-2}\,(\mathrm{MeV})^2$$

$$E=0.5141\mathrm{MeV}$$

電子動能：

$$E=K+m_0c^2$$

$$K=E-m_0c^2=0.5141\mathrm{MeV}-0.511\mathrm{MeV}=0.003\mathrm{MeV}$$

$$=3.1\mathrm{keV}$$

(b) 金鉑的功函數

$$h\nu=K+W_0$$

$$W_0=h\nu-K=\frac{hc}{\lambda}-K=\frac{12.408\times10^{-7}\,\mathrm{eV\text{-}m}}{0.71\times10^{-10}\,\mathrm{m}}-0.0031\mathrm{MeV}$$

$$=0.014376\mathrm{MeV}=14.376\mathrm{keV}$$

5.

(a) $E_{紫}=h\nu=\dfrac{hc}{\lambda}=\dfrac{12.408\times10^{-7}\,\mathrm{eV\text{-}m}}{4000\times10^{-10}\,\mathrm{m}}=3.102\mathrm{eV}$

$$E_{紅}=h\nu=\frac{hc}{\lambda}=\frac{12.408\times10^{-7}\,\mathrm{eV\text{-}m}}{7000\times10^{-10}\,\mathrm{m}}=1.772\mathrm{eV}$$

功率 $=P=40\mathrm{W}=40\mathrm{J/s}=40\times6.242\times10^{18}\,\mathrm{eV/s}$

$$=249.68\times10^{18}\,\mathrm{eV/s}$$

因此輻射率：

$$N_{紫}=\frac{249.68\times10^{18}\,\mathrm{eV/s}}{3.102\,\mathrm{eV}\text{／個}}=80.49\times10^{18}\,\text{個／ s}$$

$$N_{紅}=\frac{249.68\times10^{18}\,\mathrm{eV}}{1.772\,\mathrm{eV}\text{／個}}=140.9\times10^{18}\,\text{個／ s}$$

所以　$N_{紅}>N_{紫}$

(b) $\Delta N=N_{紅}-N_{紫}=60.41\times10^{18}$ 個／ s

6.

輻射能 $E=h\nu=\dfrac{hc}{\lambda}=\dfrac{12.408\times10^{-7}\,\text{eV-m}}{290\times10^{-9}\,\text{m}}$

$=4.28\text{eV}$

光電子的動能

$K=E-W=4.28\text{eV}-4.05\text{eV}=0.23\text{eV}$

故電位差為 0.23V

7.

(a) $\Delta\lambda=\dfrac{h}{m_pc}(1-\cos\theta)$

$(\Delta\lambda)_{\max}=2\left(\dfrac{h}{m_pc}\right)=2.66\times10^{-15}\,\text{m}$

(b) 散射光子的能量

$E'_{\max}=h\nu'=\dfrac{hc}{\lambda'}=\dfrac{hc}{\lambda+\Delta\lambda'}$

$\therefore E'_{\min}=\dfrac{\dfrac{hc}{\lambda}}{1+\dfrac{\Delta\lambda'}{\lambda}}=\dfrac{E}{1+\dfrac{\Delta\lambda'}{\lambda}}$

$=\dfrac{100\text{MeV}}{1+\dfrac{2.66\times10^{-15}\,\text{m}}{12.408\times10^{-15}\,\text{m}}}=\dfrac{100\text{MeV}}{1+0.214}=82.37\text{MeV}$

光子損失最大能量

$(\Delta E)_{\max}=E-E'_{\min}=(100-82.37)\text{MeV}$

$=17.63\text{MeV}$

8.

(a) $E_f=\dfrac{E_i}{1+\dfrac{E_i}{m_ec^2}}=\dfrac{100\,\text{keV}}{1+\dfrac{100\,\text{keV}}{0.511\times10^3\,\text{keV}}}=\dfrac{100\,\text{keV}}{1+1.957}$

$$=33.818\text{keV}$$

(b) 碰撞後電子的能量為

$$E_e = E_i - E_f = 100\text{keV} - 33.818\text{keV} = 66.182\text{keV}$$

(c) 光子動量：$P_i = \dfrac{h}{\lambda_i} = \dfrac{h}{\dfrac{hc}{E_i}} = \dfrac{E_i}{c}$

$$= 100\text{keV}/3 \times 10^8\ \text{m/s} = 33.3 \times 10^{-5}\ \text{eV-s/m}$$

電子動量：

$$E_e = \frac{P_e^2}{2m} = \frac{P_e^2 c^2}{2mc^2}$$

$$\therefore P_e c = \sqrt{2mc^2 E_e} = \sqrt{2 \times 0.511 \times 10^3 (\text{keV}) \times 66.182\text{keV}}$$

$$= 260\text{keV}$$

$$P_e = \frac{260\text{keV}}{3 \times 10^8\ \text{m/s}} = 86.66 \times 10^{-5}\ \text{eV-s/m}$$

$$\therefore \cos\phi = \frac{P_i}{P_e} = \frac{33.3 \times 10^{-5}\ \text{eV-s/m}}{86.66 \times 10^{-5}\ \text{eV-s/m}} = 0.38426$$

ϕ 約為 112.6°

9.

$$\lambda_f = \lambda_i + \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\lambda_i = \lambda_f - \frac{h}{m_0 c}(1 - \cos\theta) = \lambda_f - \frac{hc}{m_0 c^2}(1 - \cos 60°)$$

$$= 0.035\text{Å} - \frac{1}{2}\left(\frac{hc}{m_0 c^2}\right) = 0.035\text{Å} - 0.01213\text{Å}$$

$$= 0.02313\text{Å}$$

$$E_i = h\nu_i = \frac{hc}{\lambda_i} = \frac{12.408 \times 10^{-7}\ \text{eV-m}}{0.02313 \times 10^{-10}\ \text{m}}$$

$$= 5.364 \times 10^5\ \text{eV}$$

$$= 0.5364\text{MeV}$$

10.

(a) $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)\Big|_{\theta=90^\circ} = \frac{h}{m_0 c} = \frac{hc}{m_0 c^2} = 2.43\times 10^{-12}\ \text{m}$

$= 0.0243\text{Å}$

(b) 回跳電子動能

$K = E_i - E_f = \left(\frac{hc}{\lambda_i}\right)\frac{\Delta\lambda}{\lambda_i + \Delta\lambda}$

$\lambda_i = 1\text{Å}$，$\Delta\lambda = 0.243\text{Å} \quad \Rightarrow \quad K = 0.598\text{keV}$

(c) $E_i = \frac{hc}{\lambda_i} = \frac{12.408\times 10^{-7}\ \text{eV-m}}{1\times 10^{-10}\ \text{m}} = 12.408\text{keV}$

$\frac{K}{E_i} = \frac{\Delta E}{E_i} = \frac{0.598\text{keV}}{12.408\text{keV}} = 2.4\%$

11.

$E_{光子} = h\nu = E_{電子} = m_0 c^2$

光子頻率：$\nu_{光子} = \frac{m_0 c^2}{h} = \frac{0.511\times 10^6\ \text{eV}}{(6.626\times 10^{-34}\ \text{J-s})(6.242\times 10^{18}\ \text{eV/J})}$

$= 12.355\times 10^{19}/\text{s}$

光子波長：$\lambda = \frac{c}{\nu_{光子}} = \frac{3\times 10^8\ \text{m/s}}{12.355\times 10^{19}/\text{s}} = 0.0243\text{Å}$

動量：$P = \frac{E_{光子}}{c} = \frac{0.511\text{MeV}}{3\times 10^8\text{m/s}}$

$= 0.1703\times 10^{-2}\text{eV-s/m}$

12.

依動量守恆

$P_0 = P_1\cos\theta + P\cos\phi$

$0 = P_1\sin\theta - P\sin\phi$

$\tan\phi = \frac{P\sin\phi}{P\cos\phi} = \frac{P_1\sin\theta}{P_0 - P_1\cos\theta}$

$$\therefore \sin\theta = \left(\frac{P_0}{P_1} - \cos\theta\right)\tan\phi = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = \frac{\left(\frac{P_0}{P_1} - \cos\theta\right)\tan\phi}{2\sin\frac{\theta}{2}}$$

$$\cot\frac{\theta}{2} = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \frac{\left(\frac{P_0}{P_1} - \cos\theta\right)}{2\sin^2\frac{\theta}{2}}\tan\phi = \left(\frac{\frac{P_0}{P_1} - \cos\theta}{1 - \cos\theta}\right)\tan\phi$$

又 $\frac{P_0}{P_1} - \cos\theta = \frac{\lambda_1}{\lambda_0} - \cos\theta = \frac{v_0}{v_1} - \cos\theta$

$$= \frac{v_0}{v_1} - \left[1 - \frac{m_0 c}{h}(\lambda_1 - \lambda_0)\right]$$

$$= \frac{v_0}{v_1} - \left[1 - \frac{m_0 c}{h}\left(\frac{c}{v_1} - \frac{c}{v_0}\right)\right]$$

$$= \frac{v_0 - v_1}{v_1}\left[1 + \frac{m_0 c^2}{h v_0}\right]$$

$$1 - \cos\theta = \frac{m_0 c}{h}(\lambda_1 - \lambda_0) = \frac{m_0 c^2}{h}\left(\frac{1}{v_1} - \frac{1}{v_0}\right)$$

則

$$\cot\frac{\theta}{2} = \frac{\left(\frac{P_0}{P_1} - \cos\theta\right)}{1 - \cos\theta}\tan\phi = \frac{\frac{v_0 - v_1}{v_1}\left[1 + \frac{m_0 c^2}{h v_0}\right]}{\frac{m_0 c^2}{h}\left[\frac{1}{v_1} - \frac{1}{v_0}\right]}\tan\phi$$

$$= \left(1 + \frac{h v_0}{m_0 c^2}\right)\tan\phi$$

13.

(a) $\frac{K}{E} = \frac{\Delta\lambda}{\lambda + \Delta\lambda} = \frac{\frac{h}{m_0 c}(1 - \cos\theta)}{\lambda + \frac{h}{m_0 c}(1 - \cos\theta)} = \frac{\frac{2h}{m_0 c}\sin^2\frac{\theta}{2}}{\lambda + \frac{2h}{m_0 c}\sin^2\frac{\theta}{2}}$

$$=\frac{\frac{2h}{m_0c}\sin^2\frac{\theta}{2}}{\lambda\left[1+\frac{2h}{m_0c\lambda}\sin^2\frac{\theta}{2}\right]}=\frac{\frac{2h}{m_0c\lambda}\sin^2\frac{\theta}{2}}{1+\frac{2h}{m_0c\lambda}\sin^2\theta}$$

$$=\frac{\frac{2hv}{m_0c^2}\sin^2\frac{\theta}{2}}{1+\left(\frac{2hv}{m_0c^2}\right)\sin^2\frac{\theta}{2}}$$

(b) $(hv)_{光子}=0.2\text{MeV}$，$m_0c^2=0.511\text{MeV}$

$$\frac{K}{E}=\frac{E-E'}{E}=\frac{\Delta E}{E}=10\%=\frac{10}{100}=\frac{1}{10}$$

$$\therefore\frac{1}{10}=\frac{\frac{0.4\text{MeV}}{0.511\text{MeV}}\sin^2\frac{\theta}{2}}{1+\frac{0.4\text{MeV}}{0.511\text{MeV}}\sin^2\frac{\theta}{2}}\quad\Rightarrow\quad\sin\frac{\theta}{2}=0.3766$$

$\theta\simeq 44°$

14.

$$\frac{\Delta E}{E}=\frac{E-E'}{E}=\frac{K}{E}=\frac{\Delta\lambda}{\lambda+\Delta\lambda}$$

$$=\frac{1}{\lambda'}\frac{h}{m_0c}(1-\cos\theta)=\frac{hv'}{m_0c^2}(1-\cos\theta)$$

15.

(a) $E_{電子}=\frac{hc}{\lambda_c}$，$\lambda_c=\frac{hc}{E_{電子}}=\frac{12.408\times10^{-7}\text{ eV-m}}{10\times10^3\text{ keV}}$

$\lambda_c=12.408\times10^{-11}\text{ m}=1.2408\text{Å}$

(b) $E_{電子}=\frac{hc}{\lambda_{\max}}=\frac{12.408\times10^{-7}\text{ eV-m}}{0.08\times10^{-9}\text{ m}}=1.551\times10^4\text{ eV}$

$=15.51\text{keV}$

16.

$E_{電子}=\frac{hc}{\lambda_{\min}}$，$h=\frac{E_{電子}\lambda_{\min}}{c}=\frac{(40\times10^3\text{ eV})(3.11\times10^{-11}\text{ m})}{3\times10^8\text{ m/s}}$

$=41.4666\times10^{-16}$ eV-s

$=(41.4666\times10^{-16}$ eV-s$)(1.6\times10^{-19}$ J/eV$)$

$=6.6346\times10^{-34}$ J-s

17.

光子能量＝電子能量＋正電子能量，即

$E_{光子}=E_{-}+E_{+}$

$E_{-}^{2}=P_{-}^{2}c^{2}+(m_{-}c^{2})^{2}$，$E_{+}^{2}=P_{+}^{2}c^{2}+(m_{+}c^{2})^{2}$

電子於磁場中的動量為$P=eBr$

$\therefore P_{-}=eBr=(1.6\times10^{-19}\text{ coul})(0.2\text{w/m}^{2})(2.5\times10^{-2}\text{ m})$

$=8\times10^{-22}$ kg-m/s

$P_{-}c=(8\times10^{-22}\text{ kg-m/s})(3\times10^{8}\text{ m/s})=2.4\times10^{-13}\text{ J}$

$=1.5$MeV

同理，$P_{+}=P_{-}$

$P_{+}c=1.5$MeV

$\therefore E_{\pm}^{2}=P_{\pm}^{2}c^{2}+(m_{\pm}c^{2})=(1.5\text{MeV})^{2}+(0.511\text{MeV})^{2}$

$\therefore E_{\pm}=1.6$MeV

因此

$E_{光子}=E_{+}+E_{-}=3.2$MeV

波長：$\lambda_{光子}=\dfrac{hc}{E_{光子}}=\dfrac{12.408\times10^{-7}\text{ eV-m}}{3.2\times10^{6}\text{ eV}}$

$=3.9\times10^{-13}\text{ m}=0.0039\text{Å}$

18.

k_i＝起始動能，k_f＝最後動能＝0

k_1＝第一次電子減速後的動能

第一次光子能量：$h\nu_1 = k_i - k_1$

第二次光子能量：$h\nu_2 = k_1 - k_f = k_1$

$\therefore h\nu_1 + h\nu_2 = k_i$

或

$\dfrac{hc}{\lambda_1} + \dfrac{hc}{\lambda_2} = k_i$，又 $\lambda_2 = \lambda_1 + 1.3\text{Å} = \lambda_1 + \Delta\lambda$

已知 $k_i = 20\text{keV}$，$\Delta\lambda = 1.3\text{Å}$

代入得

$k_1 = 5.720\text{keV}$

$\lambda_1 = 0.869\text{Å}$，$\lambda_2 = 2.169\text{Å}$

19.

起始態：$h\nu \rightarrow |e^+ + e^- \quad \Rightarrow \quad \gamma \rightarrow e^+ + e^-$

(a) $E_{光子} = k_+ + k_- + 2m_0c^2$

$(E_{光子})_{\min} = 2m_0c^2 = P_{光子}c \quad \Rightarrow \quad P_{光子} = 2m_0c = 5.46 \times 10^{-22}\ \text{kg-m/s}$

(b) $E^2 = P^2c^2 + (m_0c^2)^2 = (K + m_0c^2)^2$

鉛原子：$P_l^2c^2 + (m_lc^2)^2 = (K_l + m_lc^2)^2$

$\therefore K_l^2 + 2K_lm_lc^2 = P_l^2c^2$

依動量守恆：$P_{光子} = P_l$

$\therefore K_l^2 + 2K_lm_lc^2 = P_{光子}^2c^2 = (E_{光子})_{\min}^2 = (2m_0c^2)^2$

$$\therefore K_l = -m_lc^2 \pm \sqrt{(m_lc^2)^2 + 4(m_0c^2)^2}$$

$$= m_lc^2\left[-1 \pm \sqrt{1 + 4\left(\frac{m_0c^2}{m_lc^2}\right)^2}\right]$$

$$= m_lc^2\left[-1 \pm \sqrt{1 + 4\left(\frac{m_0}{m_l}\right)^2}\right]$$

$$\xrightarrow[m \gg m_0]{} m_lc^2\left[-1 \pm \left(1 + \frac{1}{2}\left(\frac{2m_0}{m_l}\right)^2\right)\right]$$

取「+」：

$$K_l = m_l c^2\left[-1+\left[1+\frac{1}{2}\left(\frac{2m_0}{m_l}\right)^2\right]\right] \simeq 2\,(m_0c^2)\left(\frac{m_0}{m_l}\right)$$

m_l 為鉛原子核質量 $= M_p + M_n = (82)m_p + (124)m_n$

$\simeq 206m_p$

$$K_l = 2(0.511\text{MeV})\frac{9.1\times10^{-31}\,\text{kg}}{206\times1.67\times10^{-27}\,\text{kg}} = 5.4\text{eV}$$

第三章

1.

(a) $E = 6\text{eV}$

$$E = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2}\text{，}pc = \sqrt{2mc^2E} = 2.47\times10^3\,\text{eV}$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1.24\times10^{-6}\,\text{eV-m}}{2.47\times10^3\,\text{eV}} = 0.5\times10^{-9}\,\text{m} = 5\text{Å}$$

(b) 此為相對論能量計算

$E^2 = p^2c^2 + (m_0c^2)^2$，$E = 200\text{MeV} \gg 0.511\text{MeV}$

$\therefore E = pc$

$$\therefore \lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1.24\times10^{-6}\,\text{eV-m}}{200\times10^6} = 0.64\times10^{-14}\,\text{m}$$

$$= 0.62\times10^{-4}\,\text{Å}$$

2.

$n=1 \quad \lambda = 2d\sin\theta|_{\theta=90^\circ} = 2d = 2\times0.32\text{nm} = 0.64\times10^{-9}\,\text{m}$

$= 6.4\text{Å}$

$$p = \frac{h}{\lambda} = \frac{h}{2d}$$

$$E_{n=1} = \frac{p^2}{2m} = \frac{\left(\frac{h}{2d}\right)^2}{2m} = \frac{h^2c^2}{2mc^2(4d^2)}$$

$$=\frac{(1.24\times10^{-6}\,\text{eV-m})^2}{2\times(0.511\times10^6\,\text{eV})(4)\times(0.32\times10^{-9})^2\,\text{m}^2}$$

$$=3.67\text{eV}$$

$n=2 \quad 2\lambda=2d\sin\theta|_{\theta=90^\circ}=d$，$p=\dfrac{h}{\lambda}=\dfrac{h}{d}$

$$E_{n=2}=\frac{p^2}{2m}=\frac{\left(\frac{h}{d}\right)^2}{2m}=\frac{h^2c^2}{2mc^2d^2}=4E_1=14.68\text{eV}$$

$n=3 \quad 3\lambda=2d\sin\theta|_{\theta=90^\circ}$，$\lambda=\dfrac{2}{3}d$

$$p=\frac{h}{\lambda}=3\left(\frac{h}{2d}\right)$$

$$E_{n=3}=\frac{p^2}{2m}=\frac{9\left(\frac{h}{2d}\right)^2}{2m}=9E_1=33.03\text{eV}$$

3.

$\lambda_{\text{Na}}=\lambda_{\text{黃}}=\dfrac{h}{P_{\text{Na}}}=5890\text{Å}$，$P_{\text{Na}}=\dfrac{h}{5890d}=1.125\times10^{-27}\,\text{J-s/m}$

$$K_{\text{Na}}=\frac{P_{\text{Na}}^2}{2M}=\frac{(1.125\times10^{-27}\,\text{J-s/m})^2}{2\times9.11\times10^{-31}\,\text{kg}}=0.0695\times10^{-23}\,\text{J}$$

$$=4.34\times10^{-6}\,\text{eV}$$

4.

(a) 電子：$P_e=\dfrac{h}{\lambda}=\dfrac{6.626\times10^{-34}\,\text{J-s}}{2\times10^{-10}\,\text{m}}=3.313\times10^{-24}\,\text{J-s/m}$

光子：$P_p=\dfrac{h}{\lambda}=\dfrac{6.626\times10^{-34}\,\text{J-s}}{2\times10^{-10}\,\text{m}}=3.313\times10^{-24}\,\text{J-s/m}$

(b) 電子：$E_e=\dfrac{P_e^2}{2m}=\dfrac{P_e^2c^2}{2mc^2}=\dfrac{(6.2039\text{keV})^2}{2\times0.511\times10^6\,\text{eV}}$

$$=37.66\text{eV}$$

光子：$E_p=P_pc=(3.313\times10^{-24}\,\text{J-s/m})(3\times10^8\,\text{m/s})$

$$=9.939\times10^{-16}\,\text{J}$$

$$=6.204\text{keV}$$

(c) $\dfrac{E_p}{E_e}=\dfrac{6.204\text{keV}}{37.66\text{eV}}\simeq 164.5$

5.

$K=m_0c^2$，$\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{2m_0K}}$

$K\rightarrow K'=2K$，$\lambda'=\dfrac{h}{p'}=\dfrac{h}{\sqrt{2m_0K'}}=\dfrac{h}{\sqrt{4m_0K}}=\dfrac{1}{\sqrt{2}}\lambda$

$\therefore\lambda'=\dfrac{1}{\sqrt{2}}\times 1.7898\times 10^{-6}\,\text{Å}=1.266\times 10^{-6}\,\text{Å}$

6.

(a) $E^2=p^2c^2+(m_0c^2)^2=(K+m_0c^2)^2$，$K=$動能$=\text{eV}$

$$p^2=\frac{K^2}{c^2}+2m_0K=2m_0K\left(1+\frac{K}{2m_0c^2}\right)$$

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2m_0K}}\left(1+\frac{K}{2m_0c^2}\right)^{-\frac{1}{2}}$$

$$=\frac{h}{\sqrt{2m_0eV}}\left(1+\frac{eV}{2m_0c^2}\right)^{-\frac{1}{2}}$$

(b) 在非相對論下，$m_0c^2\gg K(\text{eV})$

$$\lambda=\frac{h}{\sqrt{2m_0eV}}=\frac{h}{\sqrt{2m_0K}}=\frac{h}{p}$$

(c) $\lambda_{rel}=\dfrac{h}{\sqrt{2m_0K}}=\left[1+\dfrac{K}{2m_0c^2}\right]^{-\frac{1}{2}}$

$$\lambda_{non\text{-}rel}=\frac{h}{\sqrt{2m_0K}}$$

$\therefore\lambda_{rel}=\dfrac{99}{100}\lambda_{non\text{-}rel}$，$\dfrac{\lambda_{rel}}{\lambda_{non\text{-}rel}}=\dfrac{1}{\left[1+\dfrac{K}{2m_0c^2}\right]^{1/2}}=\dfrac{99}{100}$

$$\therefore\frac{K}{2m_0c^2}=\left(\frac{100}{97}\right)^2-1=0.0203$$

$K=0.0406m_0c^2$

電子：$K=0.0406\times(0.511\text{MeV})=20.75\text{KeV}$

中子：$K=0.0406\times(939.6\text{MeV})=38.15\text{MeV}$

7.

$$\lambda=\frac{h}{p}=\frac{h}{m\text{v}}=\frac{h}{\dfrac{m_0\text{v}}{\sqrt{1-\beta^2}}}=\frac{h}{m_0\text{v}}\sqrt{1-\beta^2}\ \left(\beta=\frac{\text{v}}{c}\right)$$

$$=\frac{h/c}{m_0\text{v}/c}\sqrt{1-\beta^2}=\frac{h}{m_0c}\frac{\sqrt{1-\beta^2}}{\beta}=\frac{hc}{m_0c^2}\frac{\sqrt{1-\beta^2}}{\beta}$$

$$=\frac{12.408\times10^{-7}\ \text{eV-m}}{E_0}\frac{\sqrt{1-\beta^2}}{\beta}=\frac{1.24\times10^{-2}\text{MeV-Å}}{E_0(\text{MeV})}\frac{\sqrt{1-\beta^2}}{\beta}$$

8.

(a) 自由粒子德布羅意波長為

$\lambda=\dfrac{h}{p}$，或$p=\dfrac{h}{\lambda}$

$\therefore\Delta p=\dfrac{h}{\lambda^2}\Delta\lambda$

$$(\Delta x)(\Delta p)=\Delta x\left(\frac{h}{\lambda^2}\right)\Delta\lambda\geq\frac{1}{2}\hbar$$

$$\therefore(\Delta x)(\Delta\lambda)\geq\frac{h}{\lambda^2}\frac{1}{2}\hbar=\frac{\lambda^2}{4\pi}$$

(b) 由(a)得

$$(\Delta x)(\Delta\lambda)\geq\frac{\lambda^2}{4\pi}\longrightarrow(\Delta x)\left(\frac{\Delta\lambda}{\lambda}\right)\geq\frac{\lambda}{4\pi}$$

$$\therefore\Delta x=\frac{1}{\left(\dfrac{\Delta\lambda}{\lambda}\right)}\frac{\lambda}{4\pi}$$

①伽瑪射線：$\Delta x=\dfrac{1}{10^{-7}}\dfrac{5\times10^{-4}\text{Å}}{4\pi}=\dfrac{5}{4\pi}\times10^{3}\ \text{Å}$

②x射線：$\Delta x=\dfrac{1}{10^{-7}}\dfrac{5\text{Å}}{4\pi}=\dfrac{5}{4\pi}\times10^{7}\ \text{Å}$

③可見光：$\Delta x=\dfrac{1}{10^{-7}}\times\dfrac{5000\text{Å}}{4\pi}=\dfrac{5}{4\pi}\times10^{10}\,\text{Å}$

9.

$\Delta x=\lambda$，$\Delta p=\dfrac{\hbar}{2}\dfrac{1}{\Delta x}=\dfrac{\hbar}{2\lambda}=m\Delta v$

$\therefore\Delta\mathrm{v}=\dfrac{\hbar}{2m\lambda}=\dfrac{\hbar\mathrm{v}}{2mV\lambda}=\dfrac{\hbar}{2p\lambda}\mathrm{v}=\dfrac{\mathrm{v}}{4\pi}$

10.

$E=K+m_0c^2=\dfrac{p^2}{2m}+m_0c^2$

$g=\dfrac{dE}{dp}=\dfrac{p}{m}=\mathrm{v}$

或　$E^2=p^2c^2+(m_0c^2)^2$，$\dfrac{dE}{dp}=\dfrac{p}{E}c^2=\dfrac{pc^2}{mc^2}=\dfrac{p}{m}=\mathrm{v}$

11.

(a) $n\lambda=d\sin\theta$

取 $n=1$ 第一位置被偵測到

$d=\dfrac{\lambda}{\sin\theta}=\dfrac{0.5\text{Å}}{\sin5^\circ}=\dfrac{0.5\text{Å}}{0.08716}=5.736\text{Å}$

(b) 取 $n=2$ 第二位置被偵測到

$2\lambda=d\sin\theta_2\quad\sin\theta_2=\dfrac{2\lambda}{d}=2\sin5^\circ=0.17432$

$\therefore\theta_2\simeq10.05^\circ$

12.

$K=\left\langle\dfrac{1}{2}m\mathrm{v}^2\right\rangle=\dfrac{1}{2}m\langle\mathrm{v}_x^2\rangle+\dfrac{1}{2}m\langle\mathrm{v}_y^2\rangle+\dfrac{1}{2}m\langle\mathrm{v}_z^2\rangle$

$=3\times\dfrac{1}{2}m\langle\mathrm{v}_x^2\rangle=3\times\dfrac{1}{2}kT=\dfrac{3}{2}kT$

$\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{2mk}}=\dfrac{h}{\sqrt{3mkT}}$

$$m=\frac{M}{N_0}=\frac{32\text{g/mole}}{6.02\times10^{23}/\text{mole}}=5.316\times10^{-23}\text{g}$$

$$=5.316\times10^{-26}\,\text{kg}$$

$$\sqrt{3mkT}=\sqrt{3\times5.316\times10^{-26}\,\text{kg}\times1.381\times10^{-23}\text{J/K}\times300\text{K}}$$

$$=\sqrt{6.6073\times10^{-46}\,\text{kg-s}}$$

$$\lambda=\frac{h}{\sqrt{3mkT}}=\frac{6.626\times10^{-34}\,\text{J-s}}{\sqrt{6.6073\times10^{-46}\,\text{kg-s}}}=2.577\times10^{-11}\,\text{m}$$

$$=0.2577\text{Å}$$

13.

$$E_e=\frac{p^2}{2m}=\frac{p^2c^2}{2m}=\frac{E^2_{光子}}{2mc^2}=\frac{(10\times10^3\,\text{eV})^2}{2\times0.511\times10^6\,\text{eV}}=98\text{eV}$$

此動能 98eV 非常很小於運動中的能量 mc^2，則古典物理的動能為 $\frac{p^2}{2m}$。

14.

$$\lambda=\frac{h}{p}=\frac{h}{m\text{v}}=\frac{6.626\times10^{-34}\,\text{J-s}}{(0.1\times10^{-9}\,\text{kg})(0.001\times10^{-3}\,\text{m/s})}$$

$$=6.626\times10^{-18}\,\text{m}$$

$$=6.626\times10^{-3}\,\text{fm}$$

這長度相當於原子半徑等級，很顯然對於宏觀方面不太可能觀測到。

15.

$$\Delta\lambda=\frac{1}{1{,}000{,}000}\lambda=(500\times10^{-9}\,\text{m})\times10^{-6}=(5000\text{Å})\times10^{-6}$$

$$(\Delta\lambda)(\Delta x)\geq\frac{\lambda^2}{4\pi}$$

$$\Delta x\geq\frac{\lambda^2}{4\pi\Delta\lambda}=\frac{(5000\text{Å})^2}{4\pi(5000\text{Å})\times10^{-6}}=397.8\times10^6\text{Å}$$

$\simeq 0.04\text{m}$

第四章

1.

$$\rho_H = \frac{e}{\frac{4}{3}\pi R_H^2}，\rho_T = \frac{Ze}{\frac{4}{3}\pi R^3}，R^3 = ZR_H^3$$

$$R = Z^{1/3} R_H$$

2.

(a) 位能 $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{Z_Z e^2}{D} =$ 動能，$Z = Z_{Au}$，$Z = Z_\alpha$

$$D = \frac{1}{4\pi\varepsilon_0}\frac{(2)(79)(1.602\times10^{-19}\,\text{coul})^2}{5.3\text{MeV}}$$

$$= (9\times10^9\,\text{nt-m}^2/\text{coul}^2)\times\frac{(2)(79)(1.602\times10^{-19}\,\text{coul})^2}{(5.3\times10^6)(1.602\times10^{-19}\,\text{J})}$$

$$= 4.3\times10^{-14}\,\text{m}$$

(b) $\cot\dfrac{\theta}{2} = \dfrac{2b}{D}$，$b = \dfrac{1}{2}D\cot\dfrac{\theta}{2} = 3.72\times10^{-14}\,\text{m}$

3.

散射到Θ區域的粒子數（質子）

$$P = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_Z e^2}{MV^2}\right)^2 \pi\rho It\cot^2\frac{\Theta}{2}$$

$$\frac{P}{I} = \pi\rho t\left[\frac{1}{4\pi\varepsilon_0}\frac{Z_p Z_{Au}}{MV^2}e^2\right]^2\cot^2\frac{\Theta}{2}$$

(1) $\left(\dfrac{1}{4\pi\varepsilon_0}\right)^2 = (9\times10^9\,\text{nt-m}^2/\text{coul})^2 = 81\times10^{18}\,(\text{nt-m}^2/\text{coul})^2$

(2) $\rho = \dfrac{d}{M}N_0 = \dfrac{19.3\,\text{g/cm}^3}{196.967\,\text{g/mole}}\times6.023\times10^{23}/\text{mole}$

$$= 5.9\times10^{28}/\text{m}^3$$

$$(3)\left(\frac{Z_p Z_{Au} e^2}{MV^2}\right)^2=\left[\frac{(1)(79)(1.602\times10^{-19})^2}{12\times10^6\times1.602\times10^{-19}}\right]^2$$

$$=111.2286\times10^{-50}$$

$$\frac{P}{I}=2\times10^{-5}$$

$$=(81\times10^{18})(3.1416)(5.9\times10^{28})t(111.2286\times10^{-50})\cot^2 30°$$

$$=(50.1)(3)t=150.3t$$

$$\therefore t=\frac{2\times10^{-5}}{150.3}=0.01331\times10^{-5}\text{ m}\simeq 1331\text{Å}$$

4.

本題因有原子核靶的回跳關係，因此應以折合質量的粒子代替之，即

$$d=\frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{\frac{1}{2}\mu V^2}=\frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{\frac{1}{2}\left(\frac{mM}{m+M}\right)V^2}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{\frac{1}{2}MV^2}\frac{m+M}{M}$$

$$=\left(\frac{2Ze^2}{4\pi\varepsilon_0 E}\right)\left[\frac{m+M}{M}\right]$$

5.

依式（4-19），α 粒子的散射數為

$$\alpha=\pi I\rho t\left(\frac{1}{4\pi\varepsilon_0}\frac{Z_Z e^2}{MV^2}\right)^2\cot^2\frac{\Theta}{2}$$

$$E=\frac{1}{2}MV^2=\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)\left(\frac{\pi I\rho t}{\alpha}\right)^{1/2}$$

$$I=2.0\text{nA}=2\times10^{-9}\text{ coul/s}$$

$$=\frac{2\times10^{-9}\text{ coul/s}}{2\times1.6\times10^{-19}\text{ coul/}\alpha}=6.25\times10^9\ \alpha\text{/s}$$

$$\alpha=4\times10^{-4}\ \alpha\text{/s}$$

$t=0.3\mu\text{m}=0.3\times10^{-6}\,\text{m}$，$\rho=6.3\times10^{28}/\text{m}^3$

$$\Rightarrow\quad E=\frac{1}{2}MV^2=23\text{MeV}$$

6.

依式（4-16）

$$R=\frac{D}{2}\left(1+\frac{1}{\sin\dfrac{\theta}{2}}\right)，D=\frac{1}{4\pi\varepsilon_0}\frac{Z_Ze^2}{\frac{1}{2}MV^2}$$

銦原子：$Z=49$，則

$$D=(9\times10^9)\frac{(49)(1)(1.6\times10^{-19})^2}{(6\times10^6)(1.6\times10^{-19})}$$

$$=117.747\times10^{-16}\,\text{m}$$

$$\theta=60°：R=\frac{1}{2}\times117.747\times10^{-16}\,\text{m}\left(1+\frac{1}{\sin30°}\right)=176.62\times10^{-16}\,\text{m}$$

$$K.E.=6\text{MeV}-\frac{1}{4\pi\varepsilon_0}\frac{Z_Ze^2}{R}\simeq2\text{MeV}$$

$$\theta=120°：R=\frac{1}{2}\times117.747\times10^{-16}\,\text{m}\left(1+\frac{1}{\sin60°}\right)=126.813\times10^{-16}\,\text{m}$$

$$K.E.=6\text{MeV}-\frac{1}{4\pi\varepsilon_0}\frac{Z_Ze^2}{R}\simeq0.43\text{MeV}$$

7.

$$n=\frac{L}{\hbar}=\frac{2\pi L}{h}=\frac{(2)(3.1416)\times7.382\times10^{-34}\,\text{J-s}}{6.626\times10^{-34}\,\text{J-s}}=7$$

8.

波爾公式——

$$\frac{1}{\lambda}=R_H\left(\frac{1}{2^2}-\frac{1}{n^2}\right)，n=3,4,5\cdots R_H=10967757.6\text{m}^{-1}$$

巴爾麥系列：

(1) $n=3\rightarrow2$，$\dfrac{1}{\lambda_1}=R_H\left(\dfrac{1}{4}-\dfrac{1}{9}\right)$，$\lambda_1=6569.3\text{Å}$

(2) $n=4\to 2$，$\frac{1}{\lambda_2}=R_H\left(\frac{1}{4}-\frac{1}{16}\right)$，$\lambda_2=4866.0\text{Å}$

(3) $n=5\to 2$，$\frac{1}{\lambda_3}=R_H\left(\frac{1}{4}-\frac{1}{25}\right)$，$\lambda_3=4344.8\text{Å}$

9.

電子旋轉頻率為　$v=\frac{1}{T}=\frac{V}{2\pi r}$

在氫原子中

$V=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{n\hbar}$，（4-31）

$r=4\pi\varepsilon_0\frac{\hbar^2}{me^2}n^2$，（4-30）

$E=-\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{2\hbar^2 n^2}$（4-32）

因此

$$v=\frac{1}{2\pi}\left[\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{n^3\hbar^3}\right]=\frac{1}{2\pi}\left[\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{2\hbar^2 n^2}\left(\frac{2}{n\hbar}\right)\right]$$

$$=\frac{1}{2\pi}\left[\frac{2|E|}{n\hbar}\right]=\frac{2|E|}{nh}$$

10.

(a) 電子能量＝放射光子態＋第一激發態能量（$n=2$）

$=-13.6\frac{1}{4}(\text{eV})+hv$

$=-13.6\times\frac{1}{4}(\text{eV})+\frac{hc}{\lambda}$

$=-13.6\times\frac{1}{4}(\text{eV})+\frac{12.408\times 10^{-7}\text{ eV-m}}{466\text{Å}}$

$=23.2\text{eV}$

(b) 原來光子能量＝氫原子離子化能＋釋出電子能量

$=13.6\text{eV}+23.2\text{eV}$

$=36.8\text{MeV}$

11.

(a) 電流 $I=\dfrac{e}{T}=ev$

於上述第 9 題中，$v=\dfrac{2|E|}{nh}$

$\therefore I=e\left(\dfrac{2|E|}{nh}\right)$，取 $n=1$

$$I=e\frac{2|E_1|}{h}=\frac{2e(13.6\text{eV})}{h}$$

$$=\frac{2(1.602\times10^{-19})(13.6\times1.602\times10^{-19})}{6.626\times10^{-34}\text{ J-s}}$$

$$=10.535\times10^{-4}\text{ coul/s}=1.053\times10^{-3}\text{ A}$$

$$=1.053\text{mA}$$

(b) 依 Biot-Savart 定律，質子上的磁場

$B=\dfrac{\mu_0 I}{2r_1}$，$r_1\ (n=1)=5.3\times10^{-7}\text{ m}$

$$\therefore B=\frac{(4\pi\times10^{-7}\text{ T-m/A})(1.503\times10^{-3}\text{ A})}{2\times5.3\times10^{-7}\text{ m}}=1.78\times10^{-3}\text{T}\ (\text{T}=\text{Tesla})$$

12.

$$E_{電子}+E_{\text{He}}=h\nu=\frac{hc}{\lambda}=\frac{12.408\times10^{-7}\text{ eV-m}}{2400\times10^{-10}\text{ m}}=5.17\text{eV}$$

$$E_{\text{He}}=5.17\text{eV}-E_{電子}=5.17\text{eV}-3\text{eV}=2.17\text{eV}$$

$E_n=-13.6\dfrac{Z^2}{n^2}(\text{eV})=E_{\text{He}}$（$Z=2$）

$n^2=\dfrac{13.6\times4}{2.17}=25.069$，$n\simeq5$

13.

(a) 此系原子核的質量 M 為正電子質量，而等於原子質量 m，因此折合質量 μ 為

$$\mu=\frac{mM}{m+M}=\frac{mm}{m+m}=\frac{1}{2}m$$

利德堡常數 R_M 為

$$R_M=\frac{M}{M+m}R_\infty=\frac{1}{2}R_\infty$$

正電子的能階為

$$E_{正電子}=-R_M\frac{hcZ^2}{n^2}=-\frac{R_\infty hc}{2n^2}Z^2$$

因此光譜線

$$\frac{1}{\lambda}=R_M Z^2\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)=\frac{1}{2}R_\infty Z^2\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$

或

$$\frac{1}{\lambda_{正}}=\frac{1}{2}Z^2\left(\frac{1}{\lambda_{氫}}\right)$$（$Z\geq 1$ 為正-負系原子）

(b) $D_{正}=\dfrac{4\pi\varepsilon_0\hbar^2}{\mu Ze^2}n^2=2\dfrac{4\pi\varepsilon_0\hbar^2}{mZe^2}n^2=2r_H$

$D_{正}$ 為正-負電子間的距離，也是此原子系的軌道半徑，因此於任何量子數 n，$D_{正}$ 為氫原子電子軌道的兩倍大。

14.

(a) 折合質量：

$$\mu=\frac{m_{\mu^-}M_{\mu原子粒}}{m_{\mu^-}+M_{\mu原子粒}}$$

$m_{\mu^-}=207m_e$，$M_{\mu原子核}=1836m_e$

$$\mu=\frac{(207m_e)(1836m_e)}{207m_e+1836m_e}=186m_e$$

於式（4-30）μ^- 原子的原子核半徑 r 為 μ^- 原子核與 μ^- 粒子間的距離 D，因此

$$D_n=4\pi\varepsilon_0\frac{\hbar^2}{\mu Ze^2}n^2$$

取 $n=1$

$$D_1 = 4\pi\varepsilon_0 \frac{\hbar^2}{(186m_e)Ze^2} = \frac{1}{186}\left[4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2}\right]$$

$$= \frac{1}{186} r_{\text{氫原子}}(n=1) = \frac{1}{186} \times 5.3 \times 10^{-11}\,\text{m}$$

$$= 2.8 \times 10^{-13}\,\text{m} = 2.8 \times 10^{-3}\,\text{Å}$$

所以 μ^- 粒子比較靠近原子核表面

(b) 由式（4-32）

$$E_n = -\frac{\mu Z^2 e^4}{(4\pi\varepsilon_0)^2 2\hbar^2}\frac{1}{n^2}$$

取 $n=1$

$$E_1 = -\frac{186me^4}{(4\pi\varepsilon_0)^2 2\hbar^2} = -186 \times (13.6\text{eV}) = -2530\text{eV}$$

因此基態的束縛能為 2530MeV。

(c) 波長譜線

$$\frac{1}{\lambda} = R_M\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(\frac{\mu}{m_e}\right) R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= 186\, R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

第一條萊曼譜線：$n_i=2$，$n_f=1$

$$\frac{1}{\lambda} = 186\, R_\infty \left(1 - \frac{1}{4}\right) = 186(109737\text{cm}^{-1})\left(\frac{3}{4}\right)$$

$\lambda \simeq 6.5\text{Å}$

15.

於旋轉系座標為 θ，又自由旋轉的角動量 $L = I\omega =$ 定值，因此

$$\oint P_\theta\, d\theta = \oint L\, d\theta = I\omega(2\pi) = nh$$

$$I\omega = n\left(\frac{h}{2\pi}\right) = n\hbar$$

旋轉能量為

$$E=\frac{1}{2}I\omega^2=\frac{(I\omega)^2}{2I}=\frac{\hbar^2}{2I}n^2$$

16.

電子環繞軌道的圓周長 $2\pi r$ 為電子波長的整數倍，即

$2\pi r=n\lambda$，$n=1, 2, 3\cdots$

引進德布羅意波長 λ 與動量 p 間的關係

$$2\pi r=n\left(\frac{h}{p}\right)$$

$$pr=n\left(\frac{h}{2\pi}\right)=n\hbar$$

$L=n\hbar$　此為波爾量子定律

17.

依能量守恆

$(\Delta E)_i=(E_f-E_i)_i=\Delta E=h\nu$

$(\Delta E)_f=(E_f-E_i)_f$

$\therefore(\Delta E)_i=(\Delta E)_f=\Delta E=h\nu$

依動量守恆——於有原子核回跳考慮存在時，放射光子的動量等於原子核的動量，即

$$p_{光子}=\frac{E}{c}=\frac{h\nu}{c}=p_{原子核}=MV$$

於原子核回跳情形——屬於 f-state，則

$$(\Delta E)_f=h\nu_0=h\nu+\frac{1}{2}MV^2=h\nu+\frac{(MV)^2}{2M}$$

$$=h\nu+\frac{\left(\frac{h\nu}{c}\right)^2}{2M}$$

$$=h\nu+\frac{h\nu}{2Mc^2}=\Delta E+\frac{(\Delta E)^2}{2Mc^2}$$

$$\therefore h\nu_0 = \Delta E\left[1 + \frac{\Delta E}{2Mc^2}\right]$$

第五章

1.

$$[AB, C] = ABC - CAB = ABC - ACB - CAB + ACB$$
$$= A\,(BC - CB) + (AC - CA)B$$
$$= A\,[B, C] + [A, C]B$$

2.

$$\langle p_x \rangle = \int \psi^* p_x \psi d^3\boldsymbol{r} = \int \psi^*\left(-i\hbar\frac{\partial}{\partial x}\right)\psi d^3\boldsymbol{r} = -i\hbar\int \psi^*\left(\frac{\partial \psi}{\partial x}\right)d^3\boldsymbol{r}$$

$$\therefore \frac{d}{dx}\langle p_x \rangle = -i\hbar\int\left[\frac{\partial \psi^*}{\partial t}\frac{\partial \psi}{\partial x} + \psi^*\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial x}\right)\right]d^3\boldsymbol{r}$$
$$= -i\hbar\int\frac{\partial \psi^*}{\partial t}\frac{\partial \psi}{\partial x}d^3\boldsymbol{r} - i\hbar\int\psi^*\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial t}\right)d^3\boldsymbol{r}$$
$$= \int\left[-\frac{\hbar^2}{2m}\nabla^2\psi^* + V\psi^*\right]\frac{\partial \psi}{\partial x}d^3\boldsymbol{r} - \int\psi^*\frac{\partial}{\partial x}\left[-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi\right]d^3\boldsymbol{r}$$
$$= -\frac{\hbar^2}{2m}\int\nabla^2\psi^*\frac{\partial \psi}{\partial x}d^3\boldsymbol{r} + \int V\psi^*\frac{\partial \psi}{\partial x}d^3\vec{r} + \frac{\hbar^2}{2m}\int\psi^*\frac{\partial}{\partial x}\nabla^2\psi$$
$$- \int\psi^*\frac{\partial}{\partial x}V\psi d^3\boldsymbol{r}$$
$$= -\frac{\hbar^2}{2m}\left[\int\left(\nabla^2\psi^*\frac{\partial \psi}{\partial x}d^3\boldsymbol{r} - \psi^*\nabla^2\frac{\partial \psi}{\partial x}\right)d^3\boldsymbol{r}\right]$$
$$+ \left[\int V\psi^*\frac{\partial \psi}{\partial x}d^3\boldsymbol{r} - \int\psi^*\frac{\partial \psi}{\partial x}V\psi d^3\boldsymbol{r}\right]$$

第一項體積分改為面積分（Green's Theorem），即

$$\int\left[(\nabla^2\psi^*)\frac{\partial \psi}{\partial x} - \psi^*\nabla^2\left(\frac{\partial \psi}{\partial x}\right)\right]d^3\boldsymbol{r}$$
$$= \oint\left[\frac{\partial \psi}{\partial x}\nabla\psi^* - \psi^*\nabla\left(\frac{\partial \psi}{\partial x}\right)\right]\cdot d\boldsymbol{s}$$

若面積範圍很廣大時，上式的面積分會趨近於 0，因此

$$\frac{d}{dt}\langle p_x\rangle = \int V\psi^* \frac{\partial\psi}{\partial x} d^3\boldsymbol{r} - \int \psi^* \frac{\partial}{\partial x} V\psi\, d^3\boldsymbol{r}$$

$$= -\int \psi^* \frac{\partial V}{\partial x}\psi\, d^3\boldsymbol{r}$$

同理，$\frac{d}{dt}\langle p_y\rangle = -\int \psi^* \frac{\partial V}{\partial y}\psi d^3\boldsymbol{r}$

$$\frac{d}{dt}\langle p_z\rangle = -\int \psi^* \frac{\partial V}{\partial Z}\psi d^3\boldsymbol{r}$$

因此

$$\frac{d}{dt}\langle \boldsymbol{p}\rangle = -\int \psi^*(\nabla V)\psi d^3\boldsymbol{r} = -\langle \nabla V\rangle$$

3.

(1) $L_x = yp_z - zp_y$

$$[L_x, x] = [(yp_z - zp_y), x] = [yp_z, z] - [zp_y, x]$$

$$= y[p_z, x] + [y, x]p_z - z[p_y, x] - [z, x]p_y$$

$$= 0 + 0 - 0 - 0 = 0$$

(2) $[L_x, p_x] = [(yp_z - p_z x), p_x]$

$$= [yp_z, p_x] - [p_z x, p_x]$$

$$= y[p_z, p_x] + [y, p_x]p_z - p_z[x, p_x] - [p_z, p_x]x$$

$$= 0 + 0 - 0 - 0 = 0$$

(3) $[L_x, T] = \left[L_x, \frac{1}{2m}(p_x{}^2 + p_y{}^2 + p_z{}^2)\right]$

$$= \frac{1}{2m}[L_x, p_x{}^2] + \frac{1}{2m}[L_x, p_y{}^2] + \frac{1}{2m}[L_x, p_z{}^2]$$

$$[L_x, p_x{}^2] = [L_x, p_x]p_x + p_x[L_x, p_x]$$

$$= 0 + 0$$

$$[L_x, p_y{}^2] = [L_x, p_y]p_y + p_y[L_x, p_y]$$

$$= i\hbar p_z p_y + i\hbar p_y p_z = i\hbar(p_z p_y + p_y p_z)$$

$$[L_x, p_z^2] = [L_x, p_z]p_z + p_z [L_x, p_z]$$

$$= -i\hbar p_y p_z - i\hbar p_z p_y$$

$$\therefore [L_x, T] = \frac{1}{2m} [L_x, p_x^2] + \frac{1}{2m} [L_x, p_y^2] + \frac{1}{2m} [L_x, p_z^2]$$

$$= 0$$

4.

(1)（5-38-3）式

$$[L_\pm, L_z] = \mp\hbar L_\pm$$

pf：$[L_\pm, L_z] = [L_\pm \pm iL_y, L_z] = [L_x, L_z] \pm i [L_y, L_z]$

$$= -i\hbar L_y \pm i (i\hbar L_x)$$

$$= \mp\hbar (L_\pm)$$

(2)（5-38-4）式

$$[L^2, L_\pm] = 0$$

pf：$[L^2, L_\pm] = [L^2, L_x \pm iL_y]$

$$= [L^2, L_x] \pm i [L^2, L_y]$$

$$[L^2, L_x] = [L_x^2 + L_y^2 + L_z^2, L_x]$$

$$= [L_x^2, L_x] + [L_y^2, L_x] + [L_z^2, L_x]$$

$$= 0 + L_y [L_y, L_x] + [L_y, L_x]L_y + L_z [L_z, L_x] + [L_z, L_x]L_z$$

$$= 0 + L_y (-i\hbar L_z) + (-i\hbar L_z)L_y + i\hbar L_z L_y + i\hbar L_y L_z$$

$$= -i\hbar L_y L_z - i\hbar L_z L_y + i\hbar L_z L_y + i\hbar L_y L_z$$

$$= 0$$

同理，

$[L^2, L_y] = 0$，$[L^2, L_z] = 0$

$$\therefore [L^2, L_\pm] = 0$$

(3)（5-38-5）式

$$
\begin{aligned}
[L_+, L_-] &= [L_x + iL_y, L_x - iL_y] \\
&= [L_x + iL_y, L_x] - i\,[L_x + iL_y, L_y] \\
&= [L_x, L_x] + i\,[L_y, L_x] - i\,[L_x, L_y] - i\,(i)[L_y, L_y] \\
&= 0 + i\,(-i\hbar L_z) - i\,(i\hbar L_z) + 0 \\
&= \hbar L_z + \hbar L_z \\
&= 2\hbar L_z
\end{aligned}
$$

5.

自由粒子：

$E = \dfrac{p^2}{2m}$

特徵方程式為

$-\dfrac{\hbar}{2m}\dfrac{d^2\psi}{dx^2} = E\psi$

$\therefore \psi(x) = Ae^{\pm ikx}$，$k = \dfrac{1}{\hbar}\sqrt{2mE}$

邊界條件：$\psi(0) = \psi(L)$，$k = \dfrac{2n\pi}{L}$，$n = 0, \pm1, \pm2$

$\therefore k^2 = \dfrac{2mE}{\hbar^2} = \dfrac{4\pi^2 n^2}{L^2}$，$n = 0, \pm1, \pm2\cdots$

$\therefore E_n = \dfrac{\pi^2\hbar^2}{mL^2}n^2$，$n = 0, \pm1, \pm2\cdots$

E_n 為離散特徵值能譜，又

$\int_0^L \psi_n^*(x)\psi_{n'}(x)\,dx \sim \delta_{nn'}$ 為正交

6.

$E = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\,(\rho\omega)^2$

$= \dfrac{1}{2}I\omega^2$，

$$=\frac{(I\omega)^2}{2I}=\frac{L^2}{2I}$$

$I=m\rho^2$，$L=L_Z=-i\hbar\dfrac{\partial}{\partial\phi}$

因此，特徵方程式為

$$E\psi(\phi)=-\frac{L^2}{2I}\psi(\phi)$$

$$\therefore -\frac{\hbar^2}{2I}\frac{d^2\psi(\phi)}{d\phi^2}=E\psi(\phi)$$

$\therefore \psi(\phi)=Ae^{\pm i\omega\phi}$，$\omega=\dfrac{1}{\hbar}\sqrt{2IE}$

且　$\psi(0)=\psi(2\pi)$　$\Rightarrow$　$\omega=n$，$n=0,\pm1,\pm2\cdots$

特徵能量值為 $E_n=\dfrac{\hbar^2}{2I}n^2$，$n=0,\pm1,\pm2\cdots$

特徵函數為

$$\psi_n(x)\sim e^{\pm in\phi}$$

第六章

1.

$\int_{-\infty}^{\infty}\psi^*(x)\psi(x)dx=A^2\int_{-\infty}^{\infty}\left(\dfrac{\sin kx}{x}\right)^2dx=1$，$A=\dfrac{1}{\sqrt{k\pi}}$

2.

(a) $\langle x\rangle=\int_{-\infty}^{\infty}\psi^*(x)\,x\,\psi(x)dx=\left(\dfrac{\alpha}{\pi}\right)^{1/2}\int_{-\infty}^{\infty}xe^{-\alpha x^2}dx=0$

$$\langle x^2\rangle=\int_{-\infty}^{\infty}\psi^*(x)x^2\psi(x)dx=\left(\frac{\alpha}{\pi}\right)^{1/2}\int_{-\infty}^{\infty}x^2e^{-\alpha x^2}dx=\frac{1}{2\alpha}$$

$$\langle x^{17}\rangle=\int_{-\infty}^{\infty}\psi^*(x)x^{17}\psi(x)dx=\left(\frac{\alpha}{\pi}\right)^{1/2}\int_{-\infty}^{\infty}x^{17}e^{-\alpha x^2}dx=0$$

(b) $\langle p\rangle=\int_{-\infty}^{\infty}\psi^*(x)\left(-i\hbar\dfrac{\partial}{\partial x}\right)\psi(x)dx=i\alpha\hbar\left(\dfrac{\alpha}{\pi}\right)^{1/2}\int_{-\infty}^{\infty}xe^{-\alpha x^2}dx=0$

$$\langle p^2\rangle=\int\psi^*(x)\left(-\hbar^2\frac{\partial^2}{\partial x^2}\psi(x)\right)dx$$

$$= \alpha\hbar^2 \int_{-\infty}^{\infty} \psi^*(x)\psi(x)dx - \left[\alpha^2\hbar^2 \int_{-\infty}^{\infty} \psi^*(x)\, x^2\, \psi(x)dx\right]$$

$$= \alpha\hbar^2 - \alpha^2\hbar^2\left(\frac{1}{2\alpha}\right) = \frac{1}{2}\alpha\hbar^2$$

$$\Delta x = [\langle x^2 \rangle - \langle x \rangle^2]^{1/2} = \sqrt{\frac{1}{2\alpha}}$$

$$\Delta p = [\langle p^2 \rangle - \langle p \rangle^2]^{1/2} = \hbar\sqrt{\frac{\alpha}{2}}$$

$$(\Delta x)(\Delta p) = \sqrt{\frac{1}{2\alpha}}\,\hbar\sqrt{\frac{\alpha}{2}} = \frac{\hbar}{2}$$

3.

$V(x) = \frac{1}{2}kx^2$，時變性薛丁格方程式為

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,\ t)}{\partial x^2} + \frac{1}{2}kx^2\Psi\ (x,\ t) = i\hbar\frac{\partial\Psi(x,\ t)}{\partial t}$$

右邊：$i\hbar\frac{\partial\Psi(x,\ t)}{\partial t} = \frac{\hbar}{2}\sqrt{\frac{k}{m}}\Psi\ (x,\ t)$

左邊：$\frac{\partial\Psi(x,\ t)}{\partial x} = -\sqrt{\frac{km}{\hbar}}x\Psi\ (x,\ t)$

$$\frac{\partial^2\Psi(x,\ t)}{\partial x^2} = -\frac{\sqrt{km}}{\hbar}\Psi\ (x,\ t) + \frac{km}{\hbar^2}x^2\Psi\ (x,\ t)$$

因此

$$-\frac{\hbar^2}{2m}\left[-\frac{\sqrt{km}}{\hbar}\Psi\ (x,\ t) + \frac{km}{\hbar^2}x^2\Psi\ (x,\ t)\right] + \frac{1}{2}kx^2\Psi\ (x,\ t)$$

$$= \frac{1}{2}\hbar\sqrt{\frac{k}{m}}\Psi\ (x,\ t) - \frac{1}{2}kx^2\Psi\ (x,\ t) + \frac{1}{2}kx^2\Psi\ (x,\ t)$$

$$= \frac{1}{2}\hbar\sqrt{\frac{k}{m}}\Psi\ (x,\ t) = \text{右邊}$$

4.

(a) $V(x) = 0$，時變性薛丁格方程式為

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,\ t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,\ t)}{\partial t}$$

因此方程式的解為

$$\Psi\,(x,\,t) = A\sin\left(\frac{2\pi}{a}x\right)e^{-iEt/\hbar}$$

(b) 將(a)代入

$$\frac{\partial^2\Psi(x,\,t)}{\partial x^2} = -\left(\frac{2\pi}{a}\right)^2\Psi\,(x,\,t)$$

$$\frac{\partial\Psi(x,\,t)}{\partial t} = -\frac{iE}{\hbar}\Psi\,(x,\,t)$$

$$\therefore E = \frac{\hbar^2}{2m}\left(\frac{4\pi^2}{a^2}\right) = \frac{2\pi\hbar^2}{ma^2} = 4\left(\frac{\pi\hbar^2}{2ma^2}\right) = 4E_0$$

(c) $P = \int P(x)dx = \int_{-a/2}^{a/2} A^2\sin^2\left(\frac{2\pi}{a}x\right)dx = \frac{2}{a}\int_{-a/2}^{a/2}\sin^2\left(\frac{2\pi}{a}x\right)dx$

$$= \left(\frac{a}{4}\right)\int_0^{a/2}\sin^2\left(\frac{2\pi}{a}x\right)dx = \left(\frac{4}{a}\right)\left(\frac{a}{4}\right) = 1$$

5.

$$\langle x \rangle = \frac{2}{a}\int_{-a/2}^{a/2} x\sin^2\left(\frac{2\pi}{a}x\right)dx = 0$$

$$\langle x^2 \rangle = \frac{2}{a}\int_{-a/2}^{a/2} x^2\sin^2\left(\frac{2\pi}{a}x\right)dx = \frac{4}{a}\int_0^{a/2} x^2\sin^2\left(\frac{2\pi}{a}x\right)dx$$

$$= \left(\frac{a^2}{4}\right)\left(\frac{1}{3} - \frac{1}{2\pi^2}\right)$$

$$\langle p \rangle = \frac{2}{a}\int_{-a/2}^{a/2}\left(\frac{2\pi}{a}\right)(-i\hbar)\sin\left(\frac{2\pi}{a}x\right)\cos\left(\frac{2\pi}{a}x\right)dx = 0$$

$$\langle p^2 \rangle = \frac{2}{a}\int_{-a/2}^{a/2}\sin\left(\frac{2\pi}{a}x\right)\left(-\hbar^2\frac{\partial^2}{\partial x^2}\right)\sin\left(\frac{2\pi}{a}x\right)dx$$

$$= \left(\frac{2}{a}\right)\left(\frac{2\pi}{a}\right)^2\hbar^2\int_0^{a/2}\sin^2\left(\frac{2\pi}{a}x\right)dx$$

$$= \frac{4\pi^2\hbar^2}{a^2}$$

$$\Delta x = [\langle x^2 \rangle - \langle x \rangle^2]^{1/2} = \frac{a}{2}\left(\frac{1}{3} - \frac{1}{2\pi^2}\right)^{1/2}$$

$$\Delta p = [\langle p^2 \rangle - \langle p \rangle^2] = \frac{2\pi\hbar}{a}$$

$$\therefore (\Delta x)(\Delta p) = \frac{a}{2}\left(\frac{1}{3} - \frac{1}{2\pi^2}\right)^{1/2}\left(\frac{2\pi\hbar}{a}\right) = \pi\hbar\left(\frac{1}{3} - \frac{1}{2\pi^3}\right)^{1/2}$$

$$= 1.67\hbar$$

6.

(a) $\langle k \rangle = \left\langle \dfrac{p^2}{2m} \right\rangle = \dfrac{1}{2m}\langle p^2 \rangle$

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{\sqrt{km}}{\hbar}\Psi + \frac{km}{\hbar^2}x^2\Psi$$

$$\langle p^2 \rangle = \int_{-\infty}^{\infty}\Psi^*\left(-\hbar^2\frac{\partial^2}{\partial x^2}\right)\Psi dx$$

$$= \hbar\sqrt{km}\int_{-\infty}^{\infty}\Psi^*\Psi dx - km\int_{-\infty}^{\infty}\Psi^* x^2\Psi dx$$

$$= \hbar\,(km)^{1/2} - km\left(\frac{1}{(km)^{1/2}}\right)\frac{\hbar}{2} = \frac{1}{2}\hbar\sqrt{km}$$

$$\langle \mathrm{K} \rangle = \frac{1}{2m}\langle p^2 \rangle = \frac{1}{2m}\cdot\frac{1}{2}\hbar\sqrt{km} = \frac{1}{4}\hbar\sqrt{\frac{k}{m}} = \frac{1}{4}\hbar\omega$$

$$= \frac{1}{2}E\ \left(E = \frac{1}{2}\hbar\omega\right)$$

$$\langle \mathrm{V} \rangle = \left\langle \frac{1}{2}kx^2 \right\rangle = \frac{1}{2}k\int_{-\infty}^{\infty}\Psi^* x^2\Psi dx$$

$$= \frac{1}{2}k\left[\frac{1}{\sqrt{km}}\frac{1}{2}\hbar\right] = \frac{1}{4}\sqrt{\frac{k}{m}}\hbar = \frac{1}{4}\hbar\omega = \frac{1}{2}E$$

(b) 經典力學的動能

$$\mathrm{K} = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi)$$

$$\langle \mathrm{K} \rangle_T = \frac{1}{T}\int_0^T\frac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi)dt = \frac{1}{4}kA^2 = \frac{1}{2}E$$

$$\langle \mathrm{V} \rangle_T = \frac{1}{T}\int_0^T\frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)dt = \frac{1}{4}kA^2 = \frac{1}{2}E$$

因此(a)與(b)結果相同。

7.

$\Psi(x, t) = \psi(x)e^{-iEt/\hbar}$ 代入

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}(e^{-iEt/\hbar})+V(x)\psi(x)(e^{-iEt/\hbar})$$

$$=E\psi(x)e^{-iEt/\hbar}$$

$$\therefore -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

8.

光子能量 $E=pc$

光子的波函數 $\Psi(x,t)=\psi(x)e^{-iEt/\hbar}$

因此

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=\left(-i\hbar\frac{\partial}{\partial x}\right)c\Psi(x,t)$$

$$E\psi(x)=-i\hbar c\frac{\partial\psi(x)}{\partial x}$$

$$\therefore \psi(x)=e^{i\frac{\omega}{c}x}$$

則光子波函數為

$$\Psi(x,t)=e^{i\frac{\omega}{c}x}e^{-iEt/\hbar}=e^{i\frac{\omega}{c}x}e^{-i\omega t}$$

$$\therefore \Psi(x,t)=e^{i\frac{\omega}{c}(x-ct)}$$

又 $p=\hbar k$，$pc=E=\hbar kc$，$\hbar\omega=\hbar kc$，$\omega=kc$

$$\therefore \Psi(x,t)=e^{ik(x-ct)}$$

9.

非時變性薛丁格方程式：$V(x)=V_0$，$-\frac{a}{2}<x<\frac{a}{2}$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V_0\psi=E\psi$$

$$\frac{d^2\psi}{dx^2}=\frac{2m}{\hbar^2}(V_0-E)\psi$$

$$\psi(x) = Ae^{\sqrt{\frac{2m}{\hbar^2}(V_0 - E)}x} + Be^{-\sqrt{\frac{2m}{\hbar^2}(V_0 - E)}x}$$

於 $x > +\frac{a}{2}$ 時，$\psi(x)$ 存在，且趨近於 0，因此 $A = 0$

$$\therefore \psi(x) = Be^{-\frac{1}{\hbar}\sqrt{2m(V_0 - E)}x}$$

10.

(a) $\int_{-\infty}^{\infty} \Psi^* (x, t)\Psi (x, t)dx = 1$ 　得

$$A^2\left\{\int 9\psi_1^*\psi_1 dx + 16\int \psi_2^*\psi_2 dx + 12\int \psi_2^*\psi_1\, e^{i\omega t}dx + 12\int \psi_1^*\psi_2\, e^{-i\omega t}dx\right\}$$

$$= A^2(9 + 16) = 1$$

$$\therefore A = \frac{1}{5}$$

(b) $E\Psi (x, t) = i\hbar\frac{\partial}{\partial t}\left[\frac{3}{5}\psi_1 e^{-\frac{i}{2}\omega t} + \frac{4}{5}\psi_2 e^{-\frac{i}{2}3\omega t}\right]$

$$= \frac{3}{10}\hbar\omega\psi_1 e^{-\frac{i}{2}\omega t} + \frac{12}{10}\hbar\omega\psi_2 e^{-\frac{i}{2}3\omega t}$$

$$\therefore \langle E \rangle = \int \Psi^* (x, t)E\Psi (x, t)dx$$

$$= \frac{9}{50}\hbar\omega + \frac{18}{50}\hbar\omega = \frac{57}{50}\hbar\omega$$

11.

$V(x)$ 為實函數，$\Psi (x, t)$為時變性薛丁格方程式的波函數，則

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi$$

取上式的共軛，得

$$-i\hbar\frac{\partial\Psi^*}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi^*}{\partial x^2} + V(x)\Psi^*$$

又機率密度

$$P (x, t) = \Psi^* (x, t)\Psi (x, t)$$

$$\therefore \frac{\partial}{\partial t}P (x, t) = \frac{\partial\Psi^*}{\partial t}\Psi + \Psi^*\frac{\partial\Psi}{\partial t}$$

$$=\frac{1}{i\hbar}\left(\frac{\hbar^2}{2m}\right)\left(\frac{\partial^2\Psi^*}{\partial x^2}\Psi-\Psi^*\frac{\partial^2\Psi}{\partial x^2}\right)$$

$$=-\frac{\partial}{\partial x}\left[\frac{\hbar}{2mi}\left(\Psi^*\frac{\partial\Psi}{\partial x}-\frac{\partial\Psi^*}{\partial x}\Psi\right)\right]$$

$$=-\frac{\partial}{\partial x}j\,(x,t)$$

得

$$\frac{\partial P(x,t)}{\partial t}+\frac{\partial}{\partial x}j\,(x,t)=0$$

12.

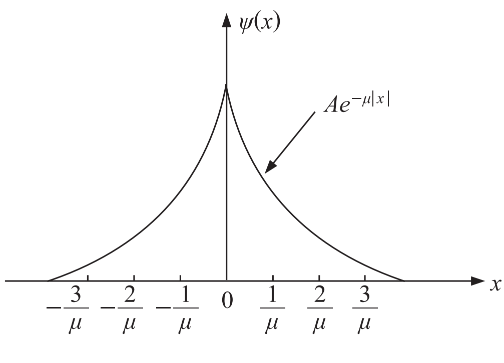
(a) 波函數　$\Psi\,(x,t)=\sqrt{\mu}\,e^{-\mu|x|}\,e^{-iEt/\hbar}$

$$P=\int_0^{\frac{1}{\mu}}\mu e^{-2\mu x}dx\text{（因為 }x\text{ 區域為＋）}$$

$$=-\frac{1}{2}\,(e^{-2}-1)=0.432$$

(b) $P=\int_{\frac{1}{\mu}}^{\frac{2}{\mu}}\mu e^{-2\mu x}dx=-\frac{1}{2}\,(e^{-1}-e^{-2})=0.059$

由此可知(a)的機率遠大於(b)的機率，這可由波函數的曲線得知



第七章

1.

$$\psi(x)=\sqrt{\frac{2}{\alpha}}\sin\left(\frac{3\pi}{\alpha}x\right)\text{，}\frac{d^2\psi}{dx^2}=-\left(\frac{3\pi}{\alpha}\right)^2\psi$$

又非時變性薛丁格方程式

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=E\psi$$

$$E=\frac{9\pi^2\hbar^2}{2md^2}$$

2.

灰塵粒子的質量

$$m=\left(\frac{4}{3}\pi r^3\right)\rho=(4\times10^{-18}\ \mathrm{m}^3)(10^4\ \mathrm{kg/m}^3)=4\times10^{-14}\ \mathrm{kg}$$

碰撞前的動能

$$K=E=\frac{1}{2}m\mathrm{v}^2=\frac{1}{2}(4\times10^{-14}\ \mathrm{kg})(10^{-2}\ \mathrm{m/s})^2=2\times10^{-18}\ \mathrm{Joule}$$

$$\Delta x=\frac{\hbar}{\sqrt{2m(V_0-E)}}=\frac{\hbar}{\sqrt{2mE}}=\frac{1.055\times10^{-34}\ \mathrm{J\text{-}s}}{\sqrt{2\times4\times10^{-14}\ \mathrm{kg}\times2\times10^{-18}\ \mathrm{Joule}}}$$

$$=2\times10^{-19}\ \mathrm{m}$$

很顯然地Δx太小了難以偵測到有穿透步階位勢。

3.

$E<V_0$，在$x\leq0$區域的行進波

$\psi(x)=$入射波$+$反射波

$$=\frac{D}{2}\left(1+\frac{ik_2}{k_1}\right)e^{ik_1x}+\frac{D}{2}\left(1-\frac{ik_2}{k_1}\right)e^{-ik_1x}$$

$$=\frac{D}{2}(e^{ik_1x}+e^{-ik_1x})+\frac{D}{2}\left(\frac{ik_2}{k_1}\right)(e^{ik_1x}-e^{-ik_1x})$$

$$=D\cos k_1x-D\left(\frac{k_2}{k_1}\right)\sin k_1x=\text{駐波}$$

4.

$$T+R=\frac{4k_1k_2}{(k_1+k_2)^2}+\left(\frac{k_2-k_1}{k_1+k_2}\right)^2$$

$$=\frac{1}{(k_1+k_2)^2}[4k_1k_2+\ (k_2-k_1)^2]=\frac{(k_1+k_2)^2}{(k_1+k_2)^2}=1$$

5.

$$k_1=\frac{1}{\hbar}\sqrt{2mE}\text{，}k_2=\frac{1}{\hbar}\sqrt{2m(E-V_0)}$$

$$\frac{k_1}{k_2}=\sqrt{\frac{E-V_0}{E}}=\sqrt{1-\frac{V_0}{E}}$$

$$R=\left(\frac{k_2-k_1}{k_1+k_2}\right)^2=\left(\frac{1-\dfrac{k_2}{k_1}}{1+\dfrac{k_2}{k_1}}\right)^2=\left(\frac{1-\sqrt{1-V_0/E}}{1+\sqrt{1-V_0/E}}\right)^2$$

$$T=\frac{4k_1k_2}{(k_1+k_2)^2}=\frac{4k_2/k_1}{\left(1+\dfrac{k_2}{k_1}\right)^2}=\frac{4\sqrt{1-V_0/E}}{(1+\sqrt{1-V_0/E})^2}$$

6.

(a) $E=2\text{eV}$，$V_0=4\text{eV}$

$$k_2a=\sqrt{\frac{2mV_0a^2}{\hbar^2}\left(1-\frac{E}{V_0}\right)}$$

$$\frac{2mV_0a^2}{\hbar^2}=\frac{2\times 9.1\times 10^{-31}\text{kg}\times 4\text{eV}\times 1.602\times 10^{-19}\text{J/eV}\times(10^{-10}\text{m})^2}{(1.055\times 10^{-34}\text{J-s})^2}$$

$$\simeq 1.05$$

$$\therefore k_2a\sqrt{(1.05)\left(1-\frac{1}{2}\right)}=0.7245$$

$$T=\left[1+\frac{\sinh^2(0.7245)}{4\left(\dfrac{2}{4}\right)\left(1-\dfrac{2}{4}\right)}\right]^{-1}=[1+\sinh^2(0.7245)]^{-1}$$

$$=0.6157$$

(b) $a=9\times 10^{-9}\,\text{m}$

$$\frac{2mV_0a^2}{\hbar^2}=8495.97$$

$$k_2 a=\sqrt{(8495.97)\left(1-\frac{1}{2}\right)}=65.176$$

$$T=\frac{1}{1+\sinh^2(65.176)}=0.9792\times10^{-56}$$

$$a=10^{-9}\ \text{m}$$

$$\frac{2mV_0a^2}{\hbar^2}=104.888，k_2a=\sqrt{104.888\left(1-\frac{1}{2}\right)}=7.242$$

$$T=\frac{1}{1+\sinh^2(7.242)}=\frac{1}{1+(698.442)^2}=2.05\times10^{-6}\ \text{m}$$

7.

公式（7-67）

$$\psi(x)=A\sin k_1x+B\cos k_1x，k_1=\frac{1}{\hbar}\sqrt{2mE}$$

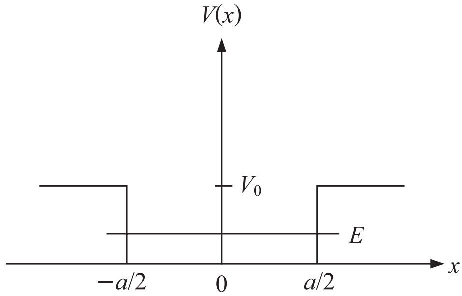
$$\frac{d^2\psi(x)}{dx^2}=-k_1^2A\sin k_1x-k_1^2B\cos k_1x=-k_1^2\psi(x)$$

$$\therefore-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}=E\psi(x)$$

為 $V(x)=0$ 的非時變性薛丁格方程式

8.

方形有限位阱為



區域 1：$x<-\frac{a}{2}$，$\psi_1(x)=ce^{k_1x}$，$k_1=\frac{1}{\hbar}\sqrt{2m(V_0-E)}$

區域 2：$-\frac{a}{2}<x<\frac{a}{2}$，（取駐波）

$$\psi_2(x)=A\cos k_2x+B\sin k_2x\text{，}k_2=\frac{1}{\hbar}\sqrt{2mE}$$

區域 3：$x>a$，$\psi_3(x)=Ge^{-k_1x}$

利用邊界條件可將 A, B, C, G 確定之。

(1) 在 $x=-\frac{a}{2}$

$$\psi_1(x)\Big|_{x=-\frac{a}{2}}=\psi_2(x)\Big|_{x=-\frac{a}{2}}\text{，}\frac{d\psi_1(x)}{dx}\Big|_{x=-\frac{a}{2}}=\frac{d\psi_2(x)}{dx}\Big|_{x=-\frac{a}{2}}$$

得

$$A=CE^{-\frac{k_1a}{2}}\left[\cos\frac{k_2a}{2}+\frac{k_1}{k_2}\sin\frac{k_2a}{2}\right]$$

$$B=-CE^{-\frac{k_1a}{2}}\left[\sin\frac{k_2a}{2}-\frac{k_1}{k_2}\cos\frac{k_2a}{2}\right]$$

$$R_{\text{左}}=\frac{V_2A^*A}{V_2B^*B}=\frac{\left[\cos\frac{k_2a}{2}+\frac{k_1}{k_2}\sin\frac{k_2a}{2}\right]^2}{\left[\sin\frac{k_2a}{2}-\frac{k_1}{k_2}\cos\frac{k_2a}{2}\right]^2}=1$$

$$\therefore\cos\frac{k_2a}{2}+\frac{k_1}{k_2}\sin\frac{k_2a}{2}=\pm\left(\sin\frac{k_2a}{2}-\frac{k_1}{k_2}\cos\frac{k_2a}{2}\right)$$

取「＋」：$\tan\frac{k_2a}{2}=\frac{1+\frac{k_1}{k_2}}{1-k_1/k_2}=\frac{k_2+k_1}{k_2-k_1}$

取「－」：$\tan\frac{k_2a}{2}=-\frac{1-k_1/k_2}{1+k_1/k_2}=-\frac{k_2-k_1}{k_2+k_1}$

因此

$$\frac{k_2+k_1}{k_2-k_1}=-\frac{k_2-k_1}{k_2+k_1}\quad\Rightarrow\quad\boxed{k_1^2+k_2^2=0}$$

(2) 在 $x=\frac{a}{2}$

$$\psi_2(x)\Big|_{x=\frac{a}{2}}=\psi_3(x)\Big|_{x=\frac{a}{2}}\text{，}\frac{d\psi_2(x)}{dx}\Big|_{x=\frac{a}{2}}=\frac{d\psi_3(x)}{dx}\Big|_{x=\frac{a}{2}}$$

得

$$A = Ge^{-\frac{k_1 a}{2}}\left[\cos\frac{k_2 a}{2} + \frac{k_1}{k_2}\sin\frac{k_2 a}{2}\right]$$

$$B = Ge^{-\frac{k_1 a}{2}}\left[\sin\frac{k_2 a}{2} - \frac{k_1}{k_2}\cos\frac{k_2 a}{2}\right]$$

$$R_{右} = \frac{V_2 B^* B}{V_2 A^* A} = \frac{\left[\sin\frac{k_2 a}{2} - \frac{k_1}{k_2}\cos\frac{k_2 a}{2}\right]^2}{\left[\cos\frac{k_2 a}{2} + \frac{k_1}{k_2}\sin\frac{k_2 a}{2}\right]^2} = 1$$

$$\therefore \sin\frac{k_2 a}{2} - \frac{k_1}{k_2}\cos\frac{k_2 a}{2} = \pm\left(\cos\frac{k_2 a}{2} + \frac{k_1}{k_2}\sin\frac{k_2 a}{2}\right)$$

取「＋」：$\tan\frac{k_2 a}{2} = \frac{k_2 + k_1}{k_2 - k_1}$

取「－」：$\tan\frac{k_2 a}{2} = \frac{k_1 - k_2}{k_1 + k_2}$

$$\therefore \frac{k_2 + k_1}{k_2 - k_1} = \frac{k_1 - k_2}{k_1 + k_2} \quad \Rightarrow \quad k_1^2 + k_2^2 = 1$$

因此若有束縛能階的話，則$k_1^2 + k_2^2 = 1$，則

$$\frac{2mV_0}{\hbar^2} = 1，或 \quad V_0 = \frac{\hbar^2}{2m}$$

故與位阱的寬度 a 無關，所以束縛能階數只有唯一的一個。

9.

(a) 中子：$m_n = 1.675 \times 10^{-27}$ kg

$$E_{中子}\ (n=1) = \frac{\pi^2\hbar^2}{2ma^2} = \frac{10.98 \times 10^{-68}\ \text{J}^2\text{-s}^2}{2 \times 1.625 \times 10^{-27} \times 10^{-28}\ \text{m}^2}$$

$$= \frac{1.0985}{3.35} \times 10^{-12}\text{J} = 2.0468 \times 10^6\text{eV}$$

$$= 2.0468\text{MeV}$$

(b) 電子

$$E_{電子} = -\frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r} = -\frac{Ae^2}{4\pi\varepsilon_0 r}，A = 原子量 = 100（取）$$

$$= -\frac{(100)(1.6\times10^{-19}\text{coul})^2}{10^{-10}\,\dfrac{\text{coul}^2}{\text{nt-m}^2}\times10^{-14}\text{m}}\times\left(\frac{1\text{eV}}{1.6\times10^{-19}\text{J}}\right)$$

$$= -10\text{MeV}$$

10.

(a) $R=\dfrac{V_0^2\sin^2(ka)}{4E(E-V_0)+V_0^2\sin^2 ka}=0$

$\therefore \sin ka=0$，$ka=n\pi$，$n=0, 1, 2, 3\cdots$

$\therefore a^2k^2=\left(\dfrac{1}{\hbar}\sqrt{2mE}\right)^2a^2=\dfrac{2mE}{\hbar^2}a^2=n^2\pi^2$

$\therefore a^2=\dfrac{\pi^2\hbar^2}{2mE}n^2$　取 $n=1$

$\therefore a^2=\dfrac{(3.14)^2(1.05\times10^{-34}\text{J-s})^2}{2\times(9.1\times10^{-31}\text{kg})(11\text{eV}\times1.6\times10^{-19}\text{J/eV})}$

$=3.431\times10^{-20}\text{m}^2$

$a=1.852\times10^{-10}\text{m}$

(b) $R=R_{\max}$ 時

$\sin ka=1$，$ka=\dfrac{\pi}{2}$

$\therefore a=\dfrac{1}{4}\dfrac{\pi^2\hbar^2}{2mE}\quad\Rightarrow\quad a=\dfrac{1}{2}\sqrt{3.431\times10^{-20}\text{m}^2}$

$\therefore a=0.92\times10^{-10}\text{m}$

11.

(a) $n^2=\dfrac{8ma^2E_n}{\pi^2\hbar^2}=\dfrac{8(9.1\times10^{-31}\text{kg})(10^{-3}\text{m})^2(0.01\text{eV}\times1.6\times10^{-19}\text{J/eV})}{(3.14)^2(1.05\times10^{-34}\text{J-s})^2}$

$=10.6\times10^8$

$n\simeq3.25\times10^4$

(b) $E_n=(\text{cons }t)n^2$，$\ln E_n=\ln(\text{cons }t)+2\ln n$

$\frac{dE}{E}=2\frac{dn}{n}$，$\therefore \frac{dn}{dE}=\frac{n}{2E}$

(1) $\frac{dn}{dE}=\frac{n}{2E}=\frac{3.25\times 10^4}{2\times 10^{-2}\text{eV}}=1.64\times 10^6$ states/eV

(2) 能量態數 $=\left(\frac{dn}{dE}\right)\frac{\text{states}}{\text{eV}}\times 10^{-4}$ eV

$=1.64\times 10^2$ states

$=164$states

12.

(a) $E_n=\frac{\pi^2\hbar^2}{8ma^2}n^2$，$n=1, 2, 3\cdots$

$E_1=\frac{(3.14)^2(1.05\times 10^{-34}\text{J-s})}{8(0.9\times 10^{-30}\text{kg})(0.5\times 10^{-9}\text{m})^2}=37.7\times 10^{-2}\text{eV}$

$E_2=4E_1=4\times 37.7\times 10^{-2}\text{eV}$

$\therefore \Delta E=E_2-E_1=(4-1)\times 37.7\times 10^{-2}\text{eV}=113.1\times 10^{-2}\text{eV}$

(b) $\Delta E=hv=\frac{hc}{\lambda}$，$\lambda=\frac{hc}{\Delta E}$

$\therefore \lambda=\frac{12.408\times 10^{-7}\text{eV-m}}{113.1\times 10^{-2}\text{eV}}=0.1097\times 10^{-5}\text{m}$

$=1097$nm

13.

(a) $E_n=\frac{\pi^2\hbar^2}{8ma^2}n^2$

$n^2=\frac{8ma^2}{\pi^2\hbar^2}E_n$

$=\frac{(8)(0.9\times 10^{-30}\text{kg})(2\times 10^{-2}\text{m})^2(1.5\text{eV}\times 1.6\times 10^{-19}\text{J/eV})}{(3.14)^2(1.05\times 10^{-34}\text{J-s})^2}$

$=63.59\times 10^{14}$

$n=7.97\times 10^{14}$

(b) $\Delta E = E_{n+1} - E_n = \left(\dfrac{\pi^2\hbar^2}{8ma^2}\right)[(n+1)^2 - n^2]$

$$= \frac{\pi^2\hbar^2}{8ma^2}(2n+1) \simeq \frac{\pi^2\hbar^2}{4ma^2}n$$

$$\therefore \Delta E = \frac{(3.14)^2(1.05\times10^{-34}\text{J-s})^2}{4(0.9\times10^{-30}\text{kg})(2\times10^{-2}\text{m})^2}\times 7.97\times10^7$$

$$= 37.55\times10^{-9}\text{eV}$$

14.

(a) $\int_{-a}^{a}\psi^*(x)\psi(x)dx = c^2\int_{-a}^{a}\left[\cos^2\left(\dfrac{\pi}{2a}x\right) + \sin^2\left(\dfrac{3\pi}{a}x\right) + \dfrac{1}{16}\cos^2\left(\dfrac{3\pi}{2a}x\right)\right.$

$$+2\cos\left(\frac{\pi}{2a}x\right)\sin\left(\frac{3\pi}{a}x\right) + \frac{1}{2}\sin\left(\frac{3\pi}{a}x\right)\cos\left(\frac{3\pi}{2a}x\right)$$

$$\left.+\frac{1}{2}\cos\left(\frac{\pi}{2a}x\right)\cos\left(\frac{3\pi}{2a}x\right)\right]dx$$

$$= c^2\left[a + a + \frac{1}{16}a + 0 + 0 + 0\right]$$

$$= \frac{33a}{16}c^2 = 1$$

$$c = \sqrt{\frac{16}{33a}}$$

(b) $\psi(x) = \sqrt{\dfrac{16}{33a}}\cos\left(\dfrac{\pi}{2a}x\right) + \sqrt{\dfrac{16}{33a}}\sin\left(\dfrac{3\pi}{a}x\right) + \dfrac{1}{4}\sqrt{\dfrac{16}{33a}}\cos\left(\dfrac{3\pi}{2a}x\right)$

無限方形位阱內的特徵函數與特徵能量值為

$$\psi_n(x) = \begin{cases} B_n\cos\left(\dfrac{n\pi}{2a}x\right)，n = 1, 3, 5\cdots \\ A_n\sin\left(\dfrac{n\pi}{2a}x\right)，n = 2, 4, 6\cdots \end{cases}$$

$E_n = \dfrac{\pi^2\hbar^2}{8ma^2}n^2$，$n = 1, 2, 3\cdots$

第一項：$\psi_1(x) = \sqrt{\dfrac{16}{33a}}\cos\left(\dfrac{\pi x}{2a}\right) \quad\Rightarrow\quad n = 1$，$E_1 = \dfrac{\pi^2\hbar^2}{8ma^2}$

第二項：$\psi_2(x) = \sqrt{\dfrac{16}{33a}}\sin\left(\dfrac{3\pi}{a}x\right) \quad\Rightarrow\quad n = 6$，$E_6 = 36E_1$

第三項：$\psi_3(x)=\sqrt{\dfrac{1}{33a}}\cos\left(\dfrac{3\pi}{2a}x\right) \quad \Rightarrow \quad n=3$，$E_3=9E_1$

(c) $\because \int_{-a}^{a}\psi^*(x)\psi(x)dx=\int_{-a}^{a}\psi_1^*\psi_1dx+\int_{-a}^{a}\psi_2^*\psi_2dx+\int_{-a}^{a}\psi_3^*\psi_3dx+$

$\int_{-a}^{a}\psi_1^*\psi_2dx+\int_{-a}^{a}\psi_2^*\psi_3dx+\int_{-a}^{a}\psi_3^*\psi_1dx+\cdots\cdots$

$=\left(\dfrac{16}{33a}\right)a+\left(\dfrac{16}{33a}\right)a+\dfrac{1}{1.6}\left(\dfrac{16}{33a}\right)a$

$=\dfrac{16}{33}+\dfrac{16}{33}+\dfrac{1}{33}=1$

$\therefore E_1\rightarrow$機率 $\quad P_1=\dfrac{16}{33}$

$E_2\rightarrow$機率 $\quad P_2=\dfrac{16}{33}$

$E_3\rightarrow$機率 $\quad P_3=\dfrac{1}{33}$

(d) $\langle E\rangle=\int\psi^*(x)E\psi dx$

$=\int_{-a}^{a}(c_1\psi_1^*+c_2\psi_2^*+c_3\psi_3^*)E_{op}(c_1\psi_1+c_2\psi_2+c_3\psi_3)\,dx$

$=c_1^2E_1+c_2^2E_2+c_3^2E_3$

$=\dfrac{\pi^2\hbar^2}{8ma^2}[c_1^2+36c_2^2+9c_3^2]$

$=\dfrac{\pi^2\hbar^2}{8ma^2}\left[\dfrac{16}{33}+\dfrac{16}{33}(36)+\dfrac{1}{33}(9)\right]$

$=\dfrac{601}{33}\left(\dfrac{\pi^2\hbar^2}{8ma^2}\right)$

15.

依非時變性薛丁格方程式解出各區域的特徵函數

$\psi_1(x)=Ae^{ik_1x}+Be^{-ik_1x}$，$k_1=\dfrac{1}{\hbar}\sqrt{2mV_0}$，$x<0$

$\psi_2(x)=Ce^{ik_2x}+De^{-ik_2x}$，$k_2=\dfrac{1}{\hbar}\sqrt{2m(9V_0)}=3k_1$，$0<x<a$

$\psi_3(x)=Fe^{ik_3x}+Ge^{-ik_3x}$，$k_3=\dfrac{1}{\hbar}\sqrt{2m(4V_0)}=2k_1$，$x>a$

$\psi_3(x)$ 中的 $G=0$，因為沒有反射波回來。

依邊界條件：

$x=0$ 處　$\psi_1(0)=\psi_2(0) \quad \rightarrow A+B=C+D$

$$\frac{d\psi_1(0)}{dx}=\frac{d\psi_2(0)}{dx}\rightarrow A-B=\frac{k_2}{k_1}(C-D)$$

$$=3\,(C-D)$$

$x=a$ 處　$\psi_2(a)=\psi_3(a)\rightarrow Ce^{ik_2a}+Be^{-ik_2a}$

$$=Fe^{ik_3a}$$

$$\left.\frac{d\psi_2}{dx}\right|_{x=a}=\left.\frac{d\psi_3}{dx}\right|_{x=a}\rightarrow Ce^{ik_2a}-Be^{-ik_2a}$$

$$=\frac{k_3}{k_2}Fe^{ik_2a}$$

得　$A=\left(\frac{5}{3}e^{ik_1a}-\frac{1}{6}e^{i5k_1a}\right)F$

$$T=\frac{V_3F^*F}{V_1A^*A}=\frac{k_3}{k_1}\frac{F^*F}{A^*A}$$

$$=2\frac{1}{\left(\frac{5}{3}e^{ik_1a}-\frac{1}{6}e^{i5k_1a}\right)^*\left(\frac{5}{3}e^{ik_1a}-\frac{1}{6}e^{i5k_1a}\right)}$$

$$=\frac{72}{101+40\cos(4k_1a)}$$

16.

(a) $\int_{-a}^{+a}\psi_2^*(x)\psi_2(x)dx=1$

$\therefore A_2^2\int_{-a}^{a}\sin^2\left(\frac{\pi}{a}x\right)dx=A_2^2\,(a)=1$，$A_2=\sqrt{\frac{1}{a}}$

$\therefore \psi_2(x)=\sqrt{\frac{1}{a}}\sin\left(\frac{\pi}{a}x\right)$

(b) $\langle x\rangle=\frac{1}{a}\int_{-a}^{a}x\sin^2\left(\frac{\pi}{a}x\right)dx=0$

$\langle x^2\rangle=\frac{1}{a}\int_{-a}^{a}x^2\sin^2\left(\frac{\pi}{a}x\right)dx=a^2\left[\frac{1}{3}-\frac{1}{2\pi^2}\right]$

$$\langle p \rangle = \int_{-a}^{a} \psi_2^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi_2 dx$$

$$= (-i\hbar)\left(\frac{1}{a}\right)\left(\frac{\pi}{a}\right)\int_{-a}^{a} \sin\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{a}x\right) dx = 0$$

$$\langle p^2 \rangle = \int_{-a}^{a} \psi_2^* \left(-\hbar^2 \frac{\partial^2}{\partial x^2}\right) \psi_2 dx$$

$$= 2 \times \frac{\pi^2\hbar^2}{a^3}\int_0^a \sin^2\left(\frac{\pi}{a}x\right) dx = \frac{\pi^2\hbar^2}{a^2}$$

(c) $\Delta x = [\langle x^2 \rangle - \langle x \rangle^2]^{1/2} = a\left[\frac{1}{3} - \frac{1}{2\pi^2}\right]^{1/2} = 0.53a$

$$\Delta p = [\langle p^2 \rangle - \langle p \rangle^2]^{1/2} = \frac{\pi\hbar}{a}$$

$$(\Delta x)(\Delta p) = \frac{\pi\hbar}{a} a\left[\frac{1}{3} - \frac{1}{2\pi^2}\right]^{1/2} = \pi\hbar\left[\frac{1}{3} - \frac{1}{2\pi^2}\right]^{1/2}$$

$$= 1.66\hbar$$

17.

$n = 2 \quad \psi_2(x) = A_2(1 - 2u^2)e^{-\frac{1}{2}u^2}$，$u^2 = \frac{\sqrt{cm}}{\hbar}x^2$

$$E_2 = \frac{5}{2}\hbar\omega = \frac{5}{2}\hbar\sqrt{\frac{c}{m}}$$

因此

$$\frac{d^2\psi_2}{dx^2} = \frac{\sqrt{cm}}{\hbar} A_2 (-2u^4 + 11u^2 - 5)e^{-\frac{1}{2}u^2}$$

代入非時變性薛丁格方程式

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_2}{dx^2} + \frac{1}{2}cx^2\psi_2$$

$$= -\frac{\hbar}{2m}\frac{\sqrt{cm}}{\hbar} A_2 (-2u^4 + 11u^2 - 5)e^{-\frac{1}{2}u^2}$$

$$+ \frac{1}{2}c\frac{\hbar}{\sqrt{cm}} u^2 A_2(1 - 2u^2)e^{-\frac{1}{2}u^2}$$

$$= \frac{\hbar}{2}\sqrt{\frac{c}{m}} A_2(5 - 10u^2)e^{-\frac{1}{2}u^2}$$

$$=\frac{5}{2}\hbar\sqrt{\frac{c}{m}}A_2(1-2u^2)e^{-\frac{1}{2}u^2}=E_2\psi_2$$

18.

(a) 簡諧振盪子的基態特徵函數為

$$\psi_0(x)=A_0e^{-\frac{1}{2}u^2}=A_0e^{-\frac{1}{2}\alpha x^2}\text{，}\alpha=\frac{\sqrt{cm}}{\hbar}$$

$$\int_{-\infty}^{\infty}\psi_0^*(x)\psi_0(x)dx=1\quad A_0=\left(\frac{\alpha}{\pi}\right)^{1/4}$$

$$\psi_0(x)=\left(\frac{\alpha}{\pi}\right)^{1/4}e^{-\frac{1}{2}\alpha x^2}$$

(b) $$\langle x\rangle=\int_{-\infty}^{\infty}\psi_0^*(x)\,x\,\psi_0(x)dx=\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty}xe^{-\alpha x^2}dx=0$$

$$\langle x^2\rangle=\int_{-\infty}^{\infty}\psi_0^*x^2\psi_0dx=2\sqrt{\frac{\alpha}{\pi}}\int_0^{\infty}x^2e^{-\alpha x^2}dx=\frac{1}{2\alpha}$$

$$\langle p\rangle=\int_{-\infty}^{\infty}\psi_0^*\left(-i\hbar\frac{\partial}{\partial x}\right)\psi_0dx=i\hbar\alpha\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty}xe^{-\alpha x^2}dx=0$$

$$\langle p^2\rangle=\int_{-\infty}^{\infty}\psi_0^*\left(-\hbar^2\frac{\partial^2}{\partial x^2}\right)\psi_0dx$$

$$=2\alpha\hbar^2\sqrt{\frac{\alpha}{\pi}}\int_0^{\infty}e^{-\alpha x^2}dx-2\alpha^2\hbar^2\sqrt{\frac{\alpha}{\pi}}\int_0^{\infty}x^2e^{-\alpha x^2}dx$$

$$=\alpha\hbar^2-\frac{1}{2}\alpha\hbar^2=\frac{1}{2}\alpha\hbar^2$$

$$\Delta x=[\langle x^2\rangle-\langle x\rangle^2]^{1/2}=\sqrt{\frac{1}{2\alpha}}$$

$$\Delta p=[\langle p^2\rangle-\langle p\rangle^2]^{1/2}=\sqrt{\frac{\alpha\hbar^2}{2}}=\hbar\sqrt{\frac{\alpha}{2}}$$

(c) $$(\Delta x)(\Delta p)=\sqrt{\frac{1}{2\alpha}}\times\hbar\sqrt{\frac{\alpha}{2}}=\frac{1}{2}\hbar$$

第八章

1.

基態能量：$E_1=-\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{\mu z^2e^4}{2\hbar^2}$　（$n=1$）

氫原子：$z=1$

$$\mu_{\mathrm{H}}=\frac{m_e m_p}{m_e+m_p}\ \left(\frac{m_e}{m_p}=5.448\times10^{-4}\right)$$

$$=0.9994m_e$$

$$\therefore E_{\mathrm{H}}=-\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{m_e e^4}{2\hbar^2}(0.99945)=(-13.6\times0.99945)\mathrm{eV}$$

$$=-13.59252\mathrm{eV}$$

氘原子：$z=1$

$$\mu_D=\frac{m_e(2m_p)}{m_e+2m_p}=0.99973m_e$$

$$\therefore E_D=(-13.6\times0.99973)\mathrm{eV}=-13.59632\mathrm{eV}$$

氦原子：$z=2$

$$\mu_{\mathrm{He}}=\frac{m_e(4m_p)}{m_e+4m_p}=0.99986m_e$$

$$E_{\mathrm{He}}=(-13.6\times0.99986)\times4=-54.3926\mathrm{eV}$$

$$\therefore\frac{E_D}{E_H}=\frac{-13.59632}{-13.59252}=1.00028$$

$$\frac{E_{\mathrm{He}}}{E_H}=\frac{-54.3926}{-13.59252}=4.00165$$

2.

(a) $E_n=(-13.59252)\dfrac{1}{n^2}\mathrm{eV}$

$n=1\quad E_1=-13.59252\mathrm{eV}$

$n=2\quad E_2=-3.3981\mathrm{eV}$

$n=3\quad E_3=-1.51028\mathrm{eV}$

(b) ①$n=3\rightarrow n=2$

$$\Delta E=h\nu=E_3-E_2=1.88785\mathrm{eV}$$

$$\nu_{32}=\frac{\Delta E}{h}=\frac{c}{\lambda_{32}}\text{，}\lambda_{32}=\frac{hc}{\Delta E}=6573\text{Å}$$

②$n=3 \rightarrow n=1$

$$\Delta E = E_3 - E_1 = 12.08224\text{eV}$$

$$\lambda_{31} = \frac{hc}{\Delta E} = 1026.98\text{Å}$$

③$n=2 \rightarrow n=1$

$$\Delta E = E_2 - E_1 = 10.19442\text{eV}$$

$$\lambda_{21} = \frac{hc}{\Delta E} = 1217.16\text{Å}$$

(c) 都屬於紅外線

3.

(a) $n=1$，$l=0$ 的基態徑向機率密度為

$$P_{10}(r) = R^*_{10}(r)\,R_{10}(r)\,4\pi r^2 \sim e^{-\frac{r}{a_0}} e^{-\frac{r}{a_0}} r^2 = r^2 e^{-\frac{2r}{a_0}}$$

$P_{10}(r)$為最大值的條件為

$$\frac{dP_{10}(r)}{dr} = 0 \quad \Rightarrow \quad r = a_0$$

(b) 徑向座標平均值——由式（8-48），$z=1$，$n=1$，$l=0$

$$\langle r \rangle_{10} = a_0\left[1 + \frac{1}{2}(1)\right] = 1.5a_0$$

(c) $\langle r \rangle_{10}$ r 值符合圖 8-4 所示。

4.

(a) $\psi_{200}(r, \theta, \varphi) = \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{1}{a_0}\right)^{3/2}\left(2 - \dfrac{r}{a_0}\right)e^{-\frac{r}{2a_0}}$（$z=1$）

$$\langle V \rangle_{200} = \frac{1}{16(2\pi)}\left(\frac{1}{a_0}\right)^3 \int \left(2 - \frac{r}{a_0}\right)e^{-\frac{r}{a_0}}\left(-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right)4\pi r^2 dr$$
$$= -\frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{4a_0}\right) = -\frac{e^2}{4\pi\varepsilon_0 a_0}\frac{1}{2^2} = -\frac{e^4}{(4\pi\varepsilon_0)^2\hbar^2 2^2} = 2E_2$$

(b) $E_2 = \dfrac{1}{2}\langle V \rangle_{200}$

(c) $E_2 = K + V$

$$\langle E_2 \rangle_{200} = \langle K \rangle_{200} + \langle V \rangle_{200} = E_2 = \frac{1}{2} \langle V \rangle_{200}$$

$$\therefore \langle K \rangle_{200} = -\frac{1}{2} \langle V \rangle_{200}$$

5.

(a) $機率 = \frac{1}{\tau} \int d\tau = \frac{1}{\frac{4}{3}\pi R^3} \int r^2 dr \sin\theta \, d\theta d\varphi$

$$= \frac{3}{4\pi R^3} \int_0^R r^2 dr \int_0^{23.5°} \sin\theta \, d\theta \int_0^{2\pi} d\varphi = 0.04147$$

$$= 4.147\%$$

(b) $n = 2$，$l = 1$，$m_l = 0$

$$\psi_{210} = \frac{1}{4\sqrt{2\pi}} \left(\frac{1}{a_0}\right)^{3/2} \left(\frac{r}{a_0}\right) e^{-\frac{r}{2a_0}} \cos\theta$$

$$\therefore P = \int \psi_{210}^* \psi_{210} d\tau$$

$$= \left[\frac{1}{4\sqrt{2\pi}} \left(\frac{1}{a_0}\right)^{3/2}\right]^2 \frac{1}{a^2} \int_0^\infty r^2 e^{-\frac{r}{a_0}} r^2 dr \int_0^{23.5°} \cos^2\theta \sin\theta \, d\theta \int_0^{2\pi} d\varphi$$

$$= 0.114376 = 11.4376\%$$

6.

$L_x = \frac{1}{2} (L_+ + L_-)$，$L_y = -\frac{i}{2} (L_+ - L_-)$

(a) $\langle l, 2|L_x|l, 1 \rangle = \frac{1}{2} \langle l, 2|L_+ + L_-|l, 1 \rangle$

$$= \frac{1}{2} \langle l, 2|L_+|l, 1 \rangle = \frac{1}{2}\hbar\sqrt{(l-1)(l+1+1)}$$

$$= \frac{\hbar}{2}\sqrt{(l-1)(l+2)}$$

$$\langle l, 2|L_y|l, 1 \rangle = -\frac{i}{2} \langle l, 2|L_+ + L_-|l, 1 \rangle$$

$$= -\frac{i}{2} \langle l, 2|L_+|l, 1 \rangle$$

$$=-\frac{1}{2}\hbar\sqrt{(l-1)(l+2)}$$

(b) $\langle\, l, 2|L_x^2|l, 0\,\rangle$

$$=\frac{1}{4}\langle\, l, 2|L_+^2+L_+L_-+L_-L_++L_-^2|l, 0\,\rangle$$

$$=\frac{1}{4}\langle\, l, 2|L_+^2|l, 0\,\rangle=\frac{\hbar^2}{4}\sqrt{l(l+1)(l-1)(l+2)}$$

$\langle\, l, 2|L_y^2|l, 0\,\rangle$

$$=-\frac{1}{4}\langle\, l, 2|L_+^2-L_-L_+-L_+L_-+L_-^2|l, 0\,\rangle$$

$$=-\frac{1}{4}\langle\, l, 2|L_+^2|l, 0\,\rangle=-\frac{1}{4}\hbar^2\sqrt{l(l+1)(l-1)(l+2)}$$

7.

(a) 已知 $Y_1^1(\theta, \varphi)$而要計算 $Y_1^0(\theta, \varphi)$，利用 L_- 算符演算，即

$$L_-Y_1^1(\theta, \varphi)=\hbar\sqrt{(1+1)(1-1+1)}\,Y_1^0(\theta, \varphi)=\sqrt{2}\hbar\,Y_1^0(\theta, \varphi)$$

則

$$-\hbar e^{-i\varphi}\left(\frac{\partial}{\partial\theta}-i\cot\theta\frac{\partial}{\partial\varphi}\right)\left[-\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\varphi}\right]=2\hbar\sqrt{\frac{3}{8\pi}}\cos\theta$$

$$\therefore Y_1^0(\theta, \varphi)=\sqrt{\frac{3}{4\pi}}\cos\theta$$

(b) 利用 L_- 算符演算 Y_1^0 可得 Y_1^{-1}，則

$$L_-Y_1^0(\theta, \varphi)=\hbar\sqrt{(1+0)(1-0+1)}\,Y_1^{-1}(\theta, \varphi)=\sqrt{2}\hbar\,Y_1^{-1}(\theta, \varphi)$$

又

$$-\hbar e^{-i\phi}\left(\frac{\partial}{\partial\theta}-i\cot\theta\frac{\partial}{\partial\varphi}\right)\left[\sqrt{\frac{3}{4\pi}}\cos\theta\right]=\hbar\sqrt{\frac{3}{4\pi}}\sin\theta\, e^{-i\varphi}$$

$$\therefore Y_1^{-1}(\theta, \varphi)=-\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{-i\varphi}$$

8.

(a) $E=\frac{1}{2}\mu \mathrm{v}^2=\frac{1}{2}\mu\,(R\omega)^2=\frac{1}{2}I\omega^2$，$I=\mu R^2$

$$=\frac{(I\omega)^2}{2I}=\frac{L^2}{2I}$$

(b) $L_{op}=L_{zop}=-i\hbar\frac{\partial}{\partial\varphi}$

$$\therefore E\Psi(\varphi, t)=i\hbar\frac{\partial\Psi(\varphi, t)}{\partial\varphi}=\frac{L_z^2}{2I}\Psi(\varphi, t)$$

$$\therefore -\frac{\hbar^2}{2I}\frac{\partial^2\Psi(\varphi, t)}{\partial\phi^2}=i\hbar\frac{\partial\Psi(\varphi, t)}{\partial t}$$

9.

(a) $\Psi(\varphi, t)=\Phi(\varphi)T(t)$代入第 8 題

$$-\frac{\hbar^2}{2I}+\frac{d^2\Phi}{d\varphi^2}=\Phi\left(i\hbar\frac{dT}{dt}\right)$$

或分離

$$-\frac{\hbar^2}{2I}\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2}=i\hbar\frac{1}{T}\frac{dT}{dt}=E$$

E 為分離常數，即總能量

因此

$$-\frac{\hbar^2}{2I}\frac{d^2\Phi}{d\varphi^2}=E\Phi$$

(b) $i\hbar\frac{1}{T}\frac{dT}{dt}=E$，$\frac{dT}{dt}=-\frac{iE}{\hbar}T$

10.

(a) $\frac{dT}{dt}=-\frac{iE}{\hbar}T$，則 $T(t)\sim e^{-\frac{iE}{\hbar}t}$

(b) 旋轉頻率 ω，則 $T(t)\sim e^{-i\omega t}$，因此

$\omega=\frac{E}{\hbar}$，$E=\hbar\omega$ 為總能量

11.

$-\frac{\hbar^2}{2I}\frac{d^2\Phi}{d\varphi^2}=E\Phi$，或 $\frac{d^2\Phi}{d\varphi^2}+\frac{2IE}{\hbar^2}\Phi=0$

$$\therefore \Phi(\varphi)=Ae^{im\varphi}+Be^{-im\varphi}$$

式中　$m=\dfrac{\sqrt{2IE}}{\hbar}$，$E=\dfrac{\hbar^2}{2I}m^2$，$m=1, 2, 3, \cdots$

12.

(a) $H=\dfrac{L_x^2+L_y^2}{2I_1}+\dfrac{L_z^2}{2I_3}=\dfrac{1}{2I_1}(L^2-L_z^2)+\dfrac{L_z^2}{2I_3}$

$$=\frac{L^2}{2I_1}+\left(\frac{1}{2I_3}-\frac{1}{2I_1}\right)L_z^2$$

$$H\psi_{nlm_l}=E_n\psi_{nlm_l}$$

$$\left[\frac{L^2}{2I_1}+\left(\frac{1}{2I_3}-\frac{1}{2I_1}\right)L_z^2\right]\psi_{nlm_l}=E_n\psi_{nlm_l}$$

$$\left[\frac{1}{2I_1}\hbar^2 l(l+1)+\left(\frac{1}{2I_3}-\frac{1}{2I_1}\right)m_l^2\hbar^2\right]=E_n$$

$$\therefore E_n=\left[\frac{l(l+1)}{2I_1}+\frac{1}{2}\left(\frac{1}{I_3}-\frac{1}{I_1}\right)m_l^2\right]\hbar^2$$

(b) $E_n=\dfrac{1}{2I_1}l(l+1)\hbar^2+\dfrac{1}{2I_1}\left(\dfrac{I_1}{I_3}-1\right)m_l^2\hbar^2$

若 $I_1\gg I_3$，則 $\dfrac{I_1}{I_3}\gg 1$，因此

$$E_n=\frac{1}{2I_1}l(l+1)\hbar^2+\frac{1}{2I_1}m_l^2\hbar^2$$

13.

$l=2$，$m_l=2, 1, 0, -1, -2$，因此有 5 個狀態，即

$|2, 2\rangle$, $|2, 1\rangle$, $|2, 0\rangle$, $|2, -1\rangle$, $|2, -2\rangle$

今特列舉 $|2, 2\rangle$ 態來演算

(a) $L_z|2, 2\rangle=2\hbar|2, 2\rangle$ ，特徵值為 $2\hbar$

(b) $\left(\dfrac{3}{5}L_x-\dfrac{4}{5}L_y\right)|2, 2\rangle$

$$=\left[\frac{3}{5}\,\frac{1}{2}(L_+-L_-)-\frac{4}{5}\left(-\frac{i}{2}\right)(L_+-L_-)\right]|2, 2\rangle$$

$$=\left[\left(\frac{8}{10}+\frac{4}{10}i\right)L_{+}+\left(\frac{6}{10}-\frac{4i}{10}\right)L_{-}\right]|2,2\rangle$$

$=A|2,3\rangle+B|2,1\rangle$ 沒有特徵值的成立。

(c) $(2L_x-6L_y+3L_z)|2,2\rangle$

$=A|2,3\rangle+B|2,1\rangle+6\hbar|2,2\rangle$ 沒有特徵值的成立。

第九章

1.

(a) $\vec{\tau}=\vec{\mu_l}\times\vec{B}\Big|_{z=8\text{cm}} \quad\Rightarrow\quad \tau=\mu_l B\sin\theta\Big|_{z=8\text{cm}}$

$\tau=(1.34\times10^{-23}\text{amp-m}^2)(0.756\text{tesla})(0.64279)$

$=6.51\times10^{-24}\,\text{nt-m}$

磁力：$F_z=\dfrac{\partial B_z}{\partial z}\mu_{lz}=\dfrac{\partial B_z}{\partial z}\mu_l\cos\theta\Big|_{z=8\text{cm}}$

$$\frac{\partial B_z}{\partial z}=0.023z\Big|_{z=8\text{cm}}=0.184\times10^{2}\text{tesla/m}$$

$F_z=(0.184\times10^{2}\text{tesla/m})(1.34\times10^{-23}\text{amp-m}^2)(0.76604)$

$=1.89\times10^{-22}\text{tesla-amp-m}=1.89\times10^{-22}\text{nt}$

(b) $\Delta E=\int\tau d\theta=\int_0^{40^\circ}\mu_l B\sin\theta d\theta=\mu_l B(1-\cos40^\circ)\Big|_{z=8\text{cm}}$

$=(1.34\times10^{-23})(0.756)(1-0.76604)$

$=0.237\times10^{-23}\text{J}$

$=1.48\times10^{-5}\text{eV}$

2.

軌道量子數 l 情況

$F_z=\dfrac{\partial B_z}{\partial z}\mu_{lz}=-\dfrac{\partial B_z}{\partial z}(g_l\mu_b m_l)$

因 $l=0\rightarrow m_l=0 \quad\Rightarrow\quad F_z=0$，$\Rightarrow$ $\dfrac{\partial B_z}{\partial z}=0$，$B_z=\text{cons}\,t$

因此電子受力與磁場梯度間的關係不適合於軌道的μ_l，那麼考慮電子自旋$\overrightarrow{\mu_s}$，即

$$F_z=\left(\frac{\partial B_z}{\partial z}\right)\mu_s=-\left(\frac{\partial B_z}{\partial z}\right)g_s\mu_b m_s$$

即 $g_s m_s=2\left(\pm\frac{1}{2}\right)=\pm 1$

$$\therefore F_z=\pm\left(\frac{\partial B_z}{\partial z}\right)\mu_b\neq 0$$

因此偏轉高度：$D=\frac{1}{2}(1\text{mm})=0.5\times 10^{-3}\ \text{m}$

$$D=\frac{1}{2}at^2=\frac{1}{2}\left(\frac{F_z}{m}\right)\left(\frac{L}{\text{v}}\right)^2=\frac{F_zL^2}{2m\text{v}^2}=\frac{\frac{1}{4}F_zL^2}{\frac{1}{2}m\text{v}^2}$$

$$=\frac{\left(\frac{\partial B_z}{\partial z}\right)L^2}{6kT}\mu_b$$

$$\therefore\frac{\partial B_z}{\partial z}=\frac{6kTD}{L^2\mu_b}=\frac{6\times(1.381\times 10^{-23}\ \text{J/K})(233\text{K})(0.5\times 10^{-3}\ \text{m})}{(50\times 10^{-2}\ \text{m})^2(9.27\times 10^{-24}\ \text{J/Tesla})}$$

$$=22.025\text{Tesla/m}$$

3.

$$D=\frac{1}{2}a_zt^2=\pm\frac{\left(\frac{\partial B_z}{\partial z}\right)\mu_b L^2}{6kT}$$

$$=\pm\frac{(1240\text{T/m})(0.927\times 10^{23}\ \text{J/Tesla})(3\times 10^{-2})^2}{6\times(1.38\times 10^{-23}\ \text{J/k})(663\text{k})}$$

$$=\pm 0.188\times 10^{-3}\ \text{m}=\pm 0.188\text{mm}$$

這上下偏離的距離為 0.38mm

4.

(a) 軌道：$\Delta E_l=g_l\mu_b m_l B$

自旋：$\Delta E_s=g_s\mu_b m_s B$

$$\Delta E = \Delta E_l + \Delta E_s = (g_l m_l + g_s m_s)\mu_b B$$
$$= (m_l + 2m_s)\mu_b B$$

(b) $n=2 \to l=0,\ 1$

(i) $l=0$，$\to m_l=0$，$m_s=\dfrac{1}{2}$，$\Delta E=\mu_b B$

$m_l=0$，$m_s=-\dfrac{1}{2}$，$\Delta E=-\mu_b B$

(ii) $l=1$，$\to m_l=1$，$m_s=\dfrac{1}{2}$，$\Delta E=2\mu_b B$

$m_s=-\dfrac{1}{2}$，$\Delta E=0$

$m_l=0$，$m_s=\dfrac{1}{2}$，$\Delta E=\mu_b B$

$m_s=-\dfrac{1}{2}$，$\Delta E=-\mu_b B$

$m_l=-l$，$m_s=\dfrac{1}{2}$，$\Delta E=0$

$m_s=-\dfrac{1}{2}$，$\Delta E=-2\mu_b B$

$n=2$ $(B=0)$

	l	m_l	m_s
$\Delta E=2\mu_b B$	1	1	$\frac{1}{2}$
$\Delta E=\mu_b B$	0	0	$\frac{1}{2}$
	1	0	$\frac{1}{2}$
$\Delta E=0$	1	-1	$\frac{1}{2}$
	1	0	$-\frac{1}{2}$
$\Delta E=-\mu_b B$	0	0	$-\frac{1}{2}$
	1	-1	$-\frac{1}{2}$
$\Delta E=-2\mu_b B$	1	0	$-\frac{1}{2}$

(c) $\Delta E = 4\,(\mu_b B) = E_2 - E_1 = -13.6\text{eV}\left(\frac{1}{4} - 1\right)$

$$= 10.2\text{eV}$$

$$\therefore B = \frac{\Delta E}{4\mu_b} = \frac{(10.2\text{eV})(1.602 \times 10^{-19}\text{J/eV})}{4 \times 0.927 \times 10^{-23}\ \text{J/tesla}}$$

$$= 4.407 \times 10^{4}\ \text{tesla}$$

5.

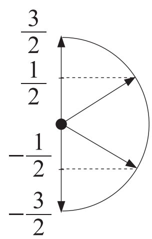
(a) $l = 1$，$j = l + \frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$

$m_j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ ①

$l = 1$，$j = l - \frac{1}{2} = 1 - \frac{1}{2} = \frac{1}{2}$

$m_j = \frac{1}{2}, -\frac{1}{2}$ ②

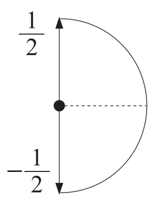
(b) a－①



$s = \frac{1}{2}$ $\uparrow$ $\uparrow$ $j = \frac{3}{2}$

$l = 1$ $\uparrow$

a－②



$l = 1$ $\uparrow$ $\downarrow$ $s = -\frac{1}{2}$

$\uparrow$ $j = \frac{1}{2}$

(c) $m_j = m_l + m_s$

①$m_l = 1$，$m_s = \frac{1}{2}$ $\rightarrow$ $m_j = \frac{3}{2}$

$m_l = 1$，$m_s = -\frac{1}{2}$ $\rightarrow$ $m_j = \frac{1}{2}$

②$m_l = 0$，$m_s = \frac{1}{2}$ $\rightarrow$ $m_j = \frac{1}{2}$

$m_l = 0$，$m_s = -\frac{1}{2}$ $\rightarrow$ $m_j = -\frac{1}{2}$

③$m_l = -1$，$m_s = \frac{1}{2} \quad \rightarrow \quad m_j = -\frac{1}{2}$

$m_l = -1$，$m_s = -\frac{1}{2} \quad \rightarrow \quad m_j = -\frac{3}{2}$

6. 躍遷規則為

$\Delta l = \pm 1$ 與$\Delta m_l = \pm 1$（不考慮LS耦合）

(a) $(2, 0, 0, \frac{1}{2}) \rightarrow (3, 1, 1, \frac{1}{2})$

$\Delta l = l - 0 = 1$，$\Delta m_l = 1 - 0 = 1$　允許躍遷

$$\Delta E = E_3 - E_2 = -13.6\text{eV}\left[\frac{1}{3^2} - \frac{1}{2^2}\right] = 1.89\text{eV}$$

相當於吸收光子

(b) $(2, 0, 0, \frac{1}{2}) \rightarrow (3, 0, 0, \frac{1}{2})$

$\Delta l = 0 - 0 = 0$，$\Delta m_l = 0 - 0 = 0$　不能允許躍遷

(c) $(4, 2, -1, \frac{1}{2}) \rightarrow (2, 1, 0, \frac{1}{2})$

$\Delta l = 1 - 2 = -1$，$\Delta m_l = 0 - (-1) = 1$，允許躍遷

但$\Delta n = 2 - 4 = 2$，$\Delta m_s = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$，不影響躍遷

$$\Delta E = E_2 - E_4 = -13.6\text{eV}\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = -2.55\text{eV}$$

相當於放射出光子

7.

$\Delta E = \pm \mu_b B$

$$\therefore (\Delta h\nu) = \Delta\left(\frac{hc}{\lambda}\right) = -\frac{hc}{\lambda^2}\Delta\lambda = \pm \mu_b B$$

$$\Delta\lambda = \frac{\lambda^2}{hc}\mu_b B$$

8.

(a) LS不耦合

$$E_n = -\frac{13.6}{4}\,\text{eV} = -3.4\text{eV}$$

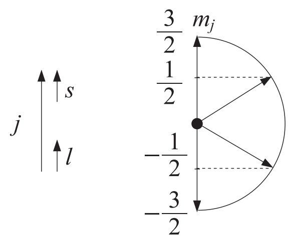
$$L = \sqrt{l(l+1)}\,\hbar = \sqrt{1(1+1)}\,\hbar = \sqrt{2}\,\hbar$$

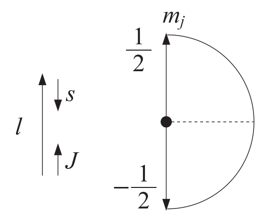
$$S = \sqrt{s(s+1)}\,\hbar = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\,\hbar = \frac{\sqrt{3}}{2}\,\hbar$$

(b) LS耦合才有予值存在

$$j_{\max} = l + s = 1 + \frac{1}{2} = \frac{3}{2}\text{，}m_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$$

$$j_{\min} = l - s = 1 - \frac{1}{2} = \frac{1}{2}\text{，}m_j = -\frac{1}{2}, \frac{1}{2}$$





(c) $\vec{\mu}_l = -\frac{g_l \mu_b}{\hbar}\vec{L}$，（$g_l = 1$）

$$\vec{\mu}_s = \frac{g_s \mu_b}{\hbar}\vec{S}\text{，（}g_s = 2\text{）}$$

$$\therefore \vec{\mu}_j = \vec{\mu}_l + \vec{\mu}_s = -\frac{\mu_b}{\hbar}(\vec{L} + 2\vec{S})$$

總角動量$\vec{J} = \vec{L} + \vec{S}$，因此$\vec{\mu}_j \neq -\vec{J}$

(d) $\vec{J} = \vec{L} + \vec{S}$，

$$J^2 = L^2 + S^2 + 2\vec{L}\cdot\vec{S} = L^2 + S^2 + 2LS\cos\theta$$

$$\cos\theta = \frac{1}{2}\frac{J^2 - L^2 - S^2}{LS} = \frac{1}{2}\frac{j(j+1) - l(l+1) - s(s+1)}{\sqrt{l(l+1)\,s\,(s+1)}}$$

(i)$j=\frac{3}{2}$，$l=1$，$s=\frac{1}{2}$，$\cos\theta=\frac{1}{\sqrt{6}}$，$\theta=\cos^{-1}\frac{1}{\sqrt{6}}$

(ii)$j=\frac{1}{2}$，$l=1$，$s=\frac{1}{2}$，$\cos\theta=-\frac{2}{\sqrt{6}}$，$\theta^{-1}=\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

9.

電子自旋磁偶矩$\overrightarrow{\mu_s}$在磁場$\vec{B}$中的交互作用位能為

$\Delta E=-\overrightarrow{\mu_s}\cdot\vec{B}$

而

$\overrightarrow{\mu_s}=-\frac{g_s\mu_b}{\hbar}\vec{S}=-2\frac{\mu_b}{\hbar}\vec{S}=-2\frac{1}{\hbar}\left(\frac{e\hbar}{2m}\right)\vec{S}$

L-S耦合時，以$\vec{J}$取代$\vec{S}$，則

$\overrightarrow{\mu_s}=-2\frac{1}{\hbar}\left(\frac{e\hbar}{2m}\right)\vec{J}$

因此在z分量（$\vec{B}$在z軸方向），

$\mu_{sz}=-2\frac{1}{\hbar}\left(\frac{e\hbar}{2m}\right)J_z$

$\therefore\Delta E=2\frac{1}{\hbar}\left(\frac{e\hbar}{2m}\right)J_zB$

$l=1$，$s=\frac{1}{2}$態：$J=1+\frac{1}{2}=\frac{3}{2}$

$(\Delta E)_{\frac{3}{2}}=2\frac{1}{\hbar}\left(\frac{e\hbar}{2m}\right)B\left(\frac{3}{2}\hbar\right)$

$l=0$，$s=\frac{1}{2}$態：$J=0+\frac{1}{2}=\frac{1}{2}$

$(\Delta E)_{\frac{1}{2}}=2\frac{1}{\hbar}\left(\frac{e\hbar}{2m}\right)B\left(\frac{1}{2}\hbar\right)$

$$\therefore\Delta E=(\Delta E)_{3/2}-(\Delta E)_{1/2}=2\frac{1}{\hbar}\left(\frac{e\hbar}{2m}\right)B\left(\frac{3}{2}-\frac{1}{2}\right)\hbar$$
$$=\left(\frac{e\hbar}{m}\right)B$$

$$\therefore B=\left(\frac{m}{e\hbar}\right)\Delta E=\frac{(9.11\times10^{-31}\,\text{kg})(0.002\text{eV})}{(1.6\times10^{-19}\,\text{coul})(6.58\times10^{-16}\,\text{eV-s})}=17\text{Tesla}$$

這內磁場相當大。

10.

(a) L-S 耦合項中的能量平均值為

$$\langle \Delta E_{LS} \rangle = \alpha_n \frac{n\{j(j+1) - l(l+1) - s(s+1)\}}{l\left(l+\frac{1}{2}\right)(l+1)}$$

$\alpha_n = \dfrac{E_n^2}{m_e c^2}$，$n = 1, 2, 3, \cdots\cdots$

$n=2$，$l=1$ 能階態中，$j = 1 + \dfrac{1}{2} = \dfrac{3}{2}$ 與 $j = 1 - \dfrac{1}{2} = \dfrac{1}{2}$，因此有兩能階態，即 $2P_{3/2}$ 與 $2P_{1/2}$。

$$\therefore \langle \Delta E_{LS} \rangle_{2P_{3/2}} = \alpha_2 \frac{2\left[\frac{3}{2}\left(\frac{3}{2}+1\right) - 1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]}{1\left(1+\frac{1}{2}\right)(1+1)}$$

$$= \alpha_2 \frac{2}{3} = \frac{2}{3}\alpha_2 = \frac{2}{3}\frac{E_2^2}{m_e c^2}$$

$$\langle \Delta E_{LS} \rangle_{2P_{1/2}} = \alpha_2 \frac{2\left[\frac{1}{2}\left(\frac{1}{2}+1\right) - 1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]}{1\left(1+\frac{1}{2}\right)(1+1)}$$

$$= -\frac{4}{3}\alpha_2 = -\frac{4}{3}\frac{E_2^2}{m_e c^2}$$

(b) $\Delta E = \langle \Delta E_{LS} \rangle_{2P_{3/2}} - \langle \Delta E_{LS} \rangle_{2P_{1/2}} = \dfrac{2}{3}\dfrac{E_2^2}{m_e c^2} - \left(-\dfrac{4}{3}\dfrac{E_2^2}{m_e c^2}\right)$

$$= 2\left(\frac{E_2^2}{m_e c^2}\right) = 0.452 \times 10^{-4}\text{eV}$$

11.

電偶矩的躍遷

$P_{fi} = \int_{-\infty}^{\infty} \psi_f^*(x)(-ex)\psi_i(x)dx$

(1) $n_l = 3 \rightarrow n_f = 0$

$$P_{03}=\int_{-\infty}^{\infty}\psi_0^*(x)(-ex)\psi_3(x)dx$$

$$=-2ec_0c_3\int_0^{\infty}(3x^2-2x^4)dx=-2ec_0c_3\left[3\frac{\sqrt{\pi}}{4}-2\left(\frac{3}{8}\sqrt{\pi}\right)\right]$$

$$=0$$

(2) $n_i=2\rightarrow n_f=0$

$$P_{02}=\int_{-\infty}^{\infty}\psi_0^*(x)(-ex)\psi_2(x)dx$$

$$=-2ec_0c_2\int_0^{\infty}(1-2x^2)\,e^{-x^2}=0$$

(3) $n_i=1\rightarrow n_f=0$

$$P_{01}=\int_{-\infty}^{\infty}\psi_0^*(x)(-ex)\psi_1(x)dx$$

$$=-2ec_1c_1\int_0^{\infty}x^2\,e^{-x^2}=\ (-2ec_0c_1)\frac{\sqrt{\pi}}{4}$$

因此只有 P_{01} 存在，即表示有躍延，則 $\Delta n=\pm1$。

12.

(a) (1) $n_i=1\rightarrow n_f=0$

$$P_{01}=\int_0^L\psi_0^*(x)(-ex)\psi_1(x)dx$$

$$=-e\left(\frac{2}{L}\right)\int_0^L x\sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{2\pi}{L}x\right)dx=\frac{16eL}{9\pi^2}$$

(2) $n_i=2\rightarrow n_f=0$

$$P_{02}=\int_0^L\psi_0^*(x)(-ex)\psi_2(x)dx$$

$$=\ (-e)\left(\frac{2}{L}\right)\int_0^L x\sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{3\pi}{L}x\right)dx=0$$

(3) $n_i=2\rightarrow n_f=1$

$$P_{12}=\int_0^L\psi_1^*(x)(-ex)\psi_2(x)dx$$

$$=\ (-e)\left(\frac{2}{L}\right)\int_0^L x\sin\left(\frac{2\pi}{L}x\right)\sin\left(\frac{3\pi}{L}x\right)dx=\frac{48e}{25\pi^2}L$$

因此，有躍遷發生情況為：

$\psi_1(x) \rightarrow \psi_0(x)$

$\psi_2(x) \rightarrow \psi_1(x)$

選擇規則為$\Delta n = \pm 1$

(b) (1) $t = t$時，波函數為

$$\Psi(x, t) = \sqrt{\frac{1}{2}}\psi_0(x)\, e^{-iE_0 t/\hbar} + \sqrt{\frac{1}{2}}\psi_1(x)\, e^{-iE_1 t/\hbar}$$

$$E_0 = \frac{\pi^2\hbar^2}{2mL^2}，E_1 = \frac{\pi^2\hbar^2}{2mL^2}(4) = 4E_0$$

因此

$$\Psi(x, t) = \sqrt{\frac{1}{2}}\psi_0(x)\, e^{-iE_0 t/\hbar} + \sqrt{\frac{1}{2}}\psi_1(x)\, e^{-i4E_0 t/\hbar}$$

(2) $\Psi^*(x, t)\Psi(x, t)$

$$= \left[\sqrt{\frac{1}{2}}\psi_0(x)\, e^{-iE_0 t/\hbar} + \sqrt{\frac{1}{2}}\psi_1(x)\, e^{-i4E_0 t/\hbar}\right]^*$$

$$\times \left[\sqrt{\frac{1}{2}}\psi_0(x)\, e^{-iE_0 t/\hbar} + \sqrt{\frac{1}{2}}\psi_1(x)\, e^{-i4E_0 t/\hbar}\right]$$

$$= \frac{1}{2}\psi_0^*\psi_0 + \frac{1}{2}\psi_1^*\psi_1 + \frac{1}{2}\psi_0^*\psi_1 e^{-3E_0 it/\hbar} + \frac{1}{2}\psi_1^*\psi_0 e^{3iE_0 t/\hbar}$$

$$\therefore P = \int_0^{L/2} \Psi^*(x, t)\Psi(x, t)$$

$$\frac{1}{2}\int_0^{L/2}\psi_0^*\psi_0 dx + \frac{1}{2}\int_0^{L/2}\psi_1^*\psi_1 dx + \frac{1}{2}\int_0^{L/2}\psi_0^*\psi_1 e^{-3iE_0 t/\hbar}dx$$

$$+ \frac{1}{2}\int_0^{L/2}\psi_1^*\psi_0 e^{3iE_0 t/\hbar}dx$$

$$= \frac{1}{2}\left(\frac{2}{L}\right)\int_0^{L/2}\sin^2\left(\frac{\pi}{L}x\right)dx + \frac{1}{2}\left(\frac{2}{L}\right)\int_0^{L/2}\sin^2\left(\frac{2\pi}{L}x\right)dx + 0 + 0$$

$$= \left(\frac{1}{L}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{L}\right)\left(\frac{1}{4}\right) = \frac{1}{2}$$

因此機率與時間無關。

13.

因 $n = 2$，$l = 0, 1$ 則

(i) $n=2$，$l=0$，$m_l=0$

(ii) $n=2$，$l=1$，$m_l=1, 0, -1$

$\psi_i\,(r, \theta, \phi)$有 4 個 states

① $\psi_{i1}\,(r, \theta, \phi)=(2, 0, 0)\rightarrow\psi_{200}=A\left(2-\dfrac{r}{a_0}\right)e^{-\frac{r}{2a_0}}$

② $\psi_{i2}\,(r, \theta, \phi)=(2, 1, 1)\rightarrow\psi_{211}=Br\,e^{-\frac{r}{2a_0}}\sin\theta\, e^{i\phi}$

③ $\psi_{i3}\,(r, \theta, \phi)=(2, 1, 0)\rightarrow\psi_{210}=Dr\,e^{-\frac{r}{2a_0}}\cos\theta$

④ $\psi_{i4}\,(r, \theta, \phi)=(2, 1, -1)\rightarrow\psi_{21-1}=Br\,e^{-\frac{r}{2a_0}}\sin\theta\, e^{-i\phi}$

末態：$n=1$，$l=0$，$m=0$，

$$\psi_f\,(r, \theta, \phi)=(1, 0, 0)\rightarrow\psi_{100}=F\,e^{-\frac{r}{a_0}}$$

因此計算 P_{fi} 有 4 個積分工作進行。

(1)$P_{fi}=\int\psi_f^*\,(r, \theta, \phi)(e\vec{r})\psi_{i1}\,(r, \theta, \phi)d\tau$

$=\int\psi_{100}^*\,(r, \theta, \phi)(e\vec{r})\psi_{200}\,(r, \theta, \phi)d\tau$

$=e\int\psi_{100}^*\,[r\sin\theta\cos\phi\,\hat{i}+r\sin\theta\sin\phi\,\hat{j}+r\cos\theta\,\hat{k}]\psi_{200}d\tau$

$=0$ 禁止躍遷

(2)$P_{fi}=\int\psi_f^*\,(e\vec{r})\psi_{i2}d\tau$

$=e\int\psi_{100}^*\,(r\sin\theta\cos\phi\,\hat{i}+r\sin\theta\sin\phi\,\hat{j}+r\cos\theta\,\hat{k})\psi_{211}d\tau$

$\neq 0$，$\Delta l=-1$

(3) $P_{fi}=\int\psi_f\,(e\vec{r})\psi_{i3}d\tau$

$=e\int\psi_{100}^*\,[r\sin\theta\cos\phi\,\hat{i}+r\sin\theta\sin\phi\,\hat{j}+r\cos\theta\,\hat{k}]\psi_{210}d\tau$

$\neq 0$，$\Delta l=1$

(4) $P_{fi}=\int\psi_f\,(e\vec{r})\psi_{i4}d\tau$

$=e\int\psi_{100}^*\,[r\sin\theta\cos\phi\,\hat{i}+r\sin\theta\sin\phi\,\hat{j}+r\cos\theta\,\hat{k}]\psi_{21-1}d\tau$

$\neq 0$，$\Delta l=-1$

因此選擇規則$\Delta l = \pm 1$ 成立。

14.

(a) $P_{fi} = \int \psi_f^* (e\vec{r})\psi_i \, d\phi$

(1) $(P_{fi})_x \sim \int \psi_f^* x \, \psi_i \, d\phi$

$$= \int e^{-im_f\phi} (L \cos\phi) e^{im_f\phi} d\phi$$

$$= A\int_0^{2\pi} e^{i(m_i - m_f + 1)\phi} d\phi + B\int_0^{2\pi} e^{i(m_i - m_f - 1)\phi} d\phi$$

$\therefore m_i - m_f - 1 \neq 0$ 及 $m_i - m_f + 1 \neq 0$

$(P_{fi})_x = 0$

$m_i - m_f - 1 = 0$，及 $m_i - m_f + 1 = 0$

$(P_{fi})_x \neq 0$　躍遷存在

因此

$\Delta m = m_i - m_f = \pm 1$

(2) 同理於$(P_{fi})_y$ 情況

(b) 依式（9-45）

$$R_{fi} = \frac{4\pi^3 v^3}{3\varepsilon_0 hc^3} P_{fi}^2$$

$$\therefore \frac{R_{12}}{R_{01}} = \frac{v_{12}^3}{v_{01}^3}\left(\frac{P_{12}}{P_{01}}\right)^2$$

而 $P_{12} = P_{01} = 2\pi$，$\Delta m = \pm 1$

且 $E_n = \dfrac{\hbar^2}{2I} m^2$，$m = 0, 1, 2, 3\cdots$

$$\Delta E = hv_{12} = \frac{\hbar^2}{2I}(4 - 1) = \frac{3}{2}\left(\frac{\hbar^2}{I}\right)$$

$$\Delta E = hv_{01} = \frac{\hbar^2}{2I}(1 - 0) = \frac{1}{2}\left(\frac{\hbar^2}{I}\right)$$

$$\therefore \frac{R_{12}}{R_{01}} = \left(\frac{\frac{3}{2}}{\frac{1}{2}}\right)^3 \left(\frac{2\pi}{2\pi}\right)^2 = 27$$

第十章

1.

(a) $E_1' = E_1^{(0)} + E_1^{(1)} = E_1^{(0)} + V_{11}$

$$E_1^{(0)} = \frac{\pi^2\hbar^2}{2ma^2}$$

$$V_{11} = \left\langle \psi_1^{(0)} \left| \frac{\delta}{a/2} \right| \psi_1^{(0)} \right\rangle = \frac{2\delta}{a}\left(\frac{2}{a}\right)\int_{-a/2}^{a/2} x\cos^2\left(\frac{\pi}{a}x\right)dx = 0$$

(b) $\psi_1'(x) = \sum\limits_{m\neq 1} \dfrac{V_{m1}}{E_1^{(0)} - E_m^{(0)}}\psi_m^{(0)} = \sum\limits_{m\neq 1} a_{1m}\,\psi_m^{(0)}(x)$

$$a_{1m} = \frac{V_{m1}}{E_1^{(0)} - E_m^{(0)}} = \frac{2ma^2}{\pi^2\hbar^2}\frac{1}{1-m^2}\int_{-a/2}^{a/2}\left(\frac{2\delta}{a}x\right)\psi_m^{(0)*}\,\psi_1^{(0)}\,dx$$

$m = 3, 5, 7\cdots$

$$a_{1m} = \frac{4m\delta a}{\pi^2\hbar^2}\frac{1}{1-m^2}\int_{-a/2}^{a/2}\left(\frac{2}{a}\right)x\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{\pi x}{a}\right)dx$$

$$= 0$$

$m = 2, 4, 6\cdots$

$$a_{1m} = \frac{4m\delta}{\pi^2\hbar^2}\frac{1}{1-m^2}\left(\frac{2}{a}\right)\int_{-a/2}^{a/2} x\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{\pi}{a}x\right)dx$$

$$= \frac{8a^2m\delta}{\pi^4\hbar^2}\frac{1}{1-m^2}\left[\frac{1}{(m+1)^2}\sin\left[\frac{1}{2}(m+1)\pi\right] + \frac{1}{(m-1)^2}\sin\left[\frac{1}{2}(m-1)\pi\right]\right]$$

$m = 2$，$a_{12} = -\dfrac{32\delta}{27\pi^2 E_1^{(0)}}$

$m = 4$，$a_{14} = \dfrac{64}{3375}\dfrac{\delta}{\pi^2 E_1^{(0)}}$

$m = 6$，$a_{16} = -\dfrac{96}{42875}\dfrac{\delta}{\pi^2 E_1^{(0)}}$

$\vdots \qquad \vdots$

$\psi_1'(x) = a_{12}\,\psi_2^{(0)}(x) + a_{14}\,\psi_4^{(0)} + \cdots\cdots$（$m \neq 1$）

因此，完整的$\psi_1'(x)$為

$$\psi_1'(x)=a_{11}\psi_1^{(0)}+a_{12}\psi_2^{(0)}(x)+a_{12}\psi_2^{(0)}(x)+a_{14}\psi_4^{(0)}(x)+\cdots$$

利用

$$\int_{-a/2}^{a/2}\psi_1'^{*}(x)\psi_1'(x)dx=1$$

得

$$a_{11}=\left[1-\left(\frac{32\delta}{27\pi^2E_1^{(0)}}\right)^2\right]^{1/2}$$

則

$$\psi_1'(x)=\sqrt{\frac{2}{a}}\left[1-\left(\frac{32\delta}{27\pi^2E_1^{(0)}}\right)^2\right]^{1/2}\cos\left(\frac{\pi}{a}x\right)$$

$$-\sqrt{\frac{2}{a}}\left(\frac{32\delta}{27\pi^2E_1^{(0)}}\right)\sin\left(\frac{2\pi}{a}x\right)+\cdots$$

2.

(a) $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2}=E\psi$

$\psi(x)=A\sin kx+B\cos kx$，$k=\sqrt{2mE}/\hbar$

邊界條件：$\psi(0)=0 \quad\Rightarrow\quad B=0$

$\psi(L)=0 \quad\Rightarrow\quad \sin kL=0$

$k_n=\dfrac{n\pi}{L}$，$n=0, 1, 2, 3\cdots$

$\psi_n(x)=\sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi}{L}x\right)$，$n=1, 2, 3\cdots$

$k_n^2=\dfrac{n^2\pi^2}{L^2}=\dfrac{2mE_n}{\hbar^2}$，$E_n=\dfrac{\pi^2\hbar^2}{2mL^2}n^2$，$n=1, 2, 3\cdots$

(b) 能量偏移

$$V_{nn}=\int_0^L\psi_n^{(0)*}(x)\left(\frac{V_0}{L}x\right)\psi_n^{(0)}(x)\,dx=\frac{1}{2}V_0$$

$$E_n'=E_n^{(0)}+V_{nn}=\frac{\pi^2\hbar^2}{2mL^2}n^2+\frac{1}{2}V_0$$

3.

(a) 中心點在 $x=0$ 的無限方位勢阱的特徵函數與特徵能量值為

$$\psi_n^{(0)}(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)，n=1, 2, 3\cdots$$

$$E_n^{(0)}=\frac{\pi^2\hbar^2}{2ma^2}n^2，n=1, 2, 3\cdots$$

第一級相關能量 $E_n^{(1)}$ 為

$$E_n^{(1)}=V_{nn}=\langle\,\psi_n^{(0)}|V(x)|\psi_n^{(0)}\,\rangle=\left\langle\,\psi_n^{(0)}\left|\alpha\delta\left(x-\frac{a}{2}\right)\right|\psi_n^{(0)}\right\rangle$$

$$=\frac{2}{a}\alpha\int_0^a\sin^2\left(\frac{n\pi}{a}x\right)\delta\left(x-\frac{2}{a}\right)dx$$

$$=\frac{\alpha}{a}[1-\ (-1)^n]$$

$$=\begin{cases}\dfrac{2\alpha}{a}，n=\text{奇數}\\[2mm] 0\quad，n=\text{偶數}\end{cases}$$

因此 $n=$偶數時，$E_n^{(1)}=0$ 即表示沒有能量偏移。

(b) $|\psi_1^{(1)}\rangle=\sum\limits_{m\neq 1}a_{1m}|\psi_m^{(0)}\rangle$

$$a_{1m}=\frac{V_{m1}}{E_1^{(0)}-E_m^{(0)}}=\frac{\langle\,\psi_m^{(0)}\,|\,V(x)\,|\,\psi_1^{(0)}\,\rangle}{E_1^{(0)}-E_m^{(0)}}$$

$$V_{1m}=\frac{2\alpha}{a}\int_0^a\sin\left(\frac{n\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)\sin\left(\frac{\pi}{a}x\right)dx$$

$$=\frac{2\alpha}{a}\sin\left(\frac{m\pi}{a}\right)$$

$m=2$，$V_{21}=0$

$m=3$，$V_{31}=-\dfrac{2\alpha}{a}$，$E_1^{(0)}-E_3^{(0)}=-8\left(\dfrac{\pi^2\hbar^2}{2ma^2}\right)$

$m=4$，$V_{41}=0$

$m=5$，$V_{51}=\dfrac{2\alpha}{a}$，$E_1^{(0)}-E_5^{(0)}=-24\left(\dfrac{\pi^2\hbar^2}{2ma^2}\right)$

$m=6$，$V_{61}=0$

$m=7$，$V_{71}=-\dfrac{2\alpha}{a}$，$E_1^{(0)}-E_7^{(0)}=-48\left(\dfrac{\pi^2\hbar^2}{2ma^2}\right)$

$$|\psi_1^{(1)}\rangle=a_{13}|\psi_3^{(0)}\rangle+a_{15}|\psi_5^{(0)}\rangle+a_{17}|\psi_7^{(0)}\rangle$$
$$=\sqrt{\frac{2}{a}}\frac{ma\alpha}{12\pi^2\hbar^2}\left[6\sin\left(\frac{3\pi}{a}x\right)-2\sin\left(\frac{5\pi}{a}x\right)+\sin\left(\frac{7\pi}{a}x\right)\right]$$

4.

薛丁格方程式為

$H^0\psi=E_0\psi$，$\Rightarrow\dfrac{L^2}{2I}\psi=E\psi$

$$-\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]=E\psi$$

令 $\psi(\theta,\phi)\sim Y_l^m(\theta,\phi)$，或$|l,m\rangle$

式中 $l=0, 1, 2, 3\cdots$

$$m=-l,-l+1,\cdots,-1,0,1,\cdots,l-1,l$$

同時

$$L^2Y_l^m(\theta,\phi)=l(l+1)\hbar^2Y_l^m(\theta,\phi)$$
$$L_zY_l^m(\theta,\phi)=m\hbar Y_l^m(\theta,\phi)$$

能量　$E_l=\dfrac{1}{2I}l(l+1)\hbar^2$，$l=0, 1, 2, \cdots$

$$l=1，E_1^{(0)}=\frac{\hbar^2}{I}$$

能量偏移為

$E_1^{(1)}=\langle lm|E\cos\theta|lm\rangle$ graund state

因此 $l=1\rightarrow m=-1, 0, 1$

(1) $E_1^{(1)}=\langle 1,-1|E\cos\theta|1,-1\rangle=0$

(2) $E_1^{(1)}=\langle 1,1|E\cos\theta|1,1\rangle=0$

(3) $E_1^{(1)}=\langle 1,0|E\cos\theta|1,0\rangle=0$

第一級相關能量 $E_1^{(1)}=0$，因此進入第二級相關能量計算

$$E_1^{(2)} = \sum_{m\neq 1} \frac{|\langle \psi_m^{(0)} | V(x) | \psi_1^{(0)} \rangle|^2}{E_1^{(0)} - E_m^{(0)}}$$

(1) $E_1^{(0)} - E_m^{(0)} = E_1^{(0)} - E_l^{(0)} = \dfrac{\hbar^2}{I} - \dfrac{1}{2I}\, l\,(l+1)\hbar^2$

$$= \frac{\hbar^2}{I}\left[1 - \frac{1}{2}l(l+1)\right]$$

(2) $\langle \psi_m^{(0)} | V(x) | \psi_1^{(0)} \rangle =$

於此計算：$\langle l_1, m_1 |\mathrm{v}| l_2, m_2 \rangle$ 有結果條件為

$m_1 = m_2,\ l_1 = l_2 \pm 1$

因此

$$\langle l, m|\cos\theta|l-1, m\rangle = \langle l-1|\cos\theta|l, m\rangle$$

$$= \left(\frac{l^2 - m^2}{4l^2 - 1}\right)^{1/2}$$

今 $\psi_1^{(0)}$ 的 states 有$|1, 1\rangle$，$|1, 0\rangle$及$|1, -1\rangle$

$\psi_m^{(0)}$ 的 states 有（$l = 2$）

$|2, 2\rangle, |2, 1\rangle, |2, 0\rangle, |2, -1\rangle, |2, -2\rangle$

因此

$$\langle \psi_m^{(0)}|\cos\theta|\psi_1^{(0)}\rangle = \langle 2, 1|\cos\theta|1, 1\rangle = \left(\frac{2^2 - 1^2}{4(2)^2 - 1}\right)^{1/2} = \sqrt{\frac{1}{5}}$$

$$\langle \psi_m^{(0)}|\cos\theta|\psi_1^{(0)}\rangle = \langle 2, 0|\cos\theta|1, 0\rangle = \left(\frac{2^2 - 0}{4(2)^2 - 1}\right)^{1/2} = \sqrt{\frac{4}{15}}$$

$$\langle \psi_m^{(0)}|\cos\theta|\psi_1^{(0)}\rangle = \langle 2, -1|\cos\theta|1, -1\rangle = \left(\frac{2^2 - (-1)^2}{4(2)^2 - 1}\right)^{1/2} = \sqrt{\frac{1}{5}}$$

$$E_1^{(2)} = \frac{E^2|\langle 2, 1|\cos\theta|1, 1\rangle|^2}{E_1^{(0)} - E_2^{(0)}} + \frac{E^2|\langle 2, 0|\cos\theta|1, 0\rangle|^2}{E_1^{(0)} - E_2^{(0)}}$$

$$+ \frac{E^2|\langle 2, -1|\cos\theta|1, -1\rangle|^2}{E_1^{(0)} - E_2^{(0)}}$$

$$= -\frac{E^2 I}{2\hbar^2}\left[\frac{1}{5} + \frac{4}{15} + \frac{1}{5}\right] = -\frac{E^2 I}{2\hbar^2}\left(\frac{2}{3}\right)$$

$$= -\frac{E^2 I}{3\hbar^2}$$

5.

(a) $H^0=\frac{1}{2}m\text{v}^2=\frac{L_z^2}{2I}$，　$L_z=I\omega$

$-\frac{\hbar^2}{2\omega I}\frac{d^2}{d\phi^2}\psi=E\psi,\quad I=mR^2$

(b) $\frac{d^2\psi}{d\phi^2}+\frac{2EI}{\hbar^2}\psi=0$

$\therefore\psi(\phi)=Ae^{im\phi}$

邊界條件：$\psi(\phi)=\psi(\phi+2\pi)$

$\therefore e^{im\phi}=Ae^{im(\phi+2\pi)}$

$\therefore m=0,\pm1,\pm2,\cdots$

同時，利用

$\int_0^{2\pi}\psi^*(\phi)\psi(\phi)d\phi=1\quad\Rightarrow A=\sqrt{\frac{1}{2\pi}}$

因此，特徵函數為

$\psi_m(\phi)=\sqrt{\frac{1}{2\pi}}e^{im\phi}$，$m=0,\pm1,\pm2\cdots$

(c) $L_z\psi_m(\phi)=-i\hbar\frac{\partial}{\partial\phi}(Ae^{im\phi})=m\hbar\psi_m(\phi)$

$L_z=m\hbar$

(d) $H^0\psi_m(\phi)=E_m\psi_m(\phi)\Rightarrow\frac{L_z^2}{2I}\psi_m(\phi)=E_m\psi_m(\phi)$

$\therefore E_m=\frac{\hbar^2}{2I}m^2$，$m=0,\pm1,\pm2\cdots$

(e) $V(x)=-q\varepsilon x$

$$E_1^{(1)}=\langle\psi_m^{(0)}|V(x)|\psi_m^{(0)}\rangle=\langle\psi_0^{(0)}|-qEx|\psi_0^{(0)}\rangle$$

$$=-qE\left(\frac{1}{2\pi}\right)\int_0^{2\pi}xd\phi=0$$

$\therefore E_1'=E_1^{(0)}+E_1^{(1)}=E_1^{(0)}=\frac{\hbar^2}{2I}$

(f) 第一級相關特徵函數（基態）

$$\psi_0^{(1)}(x)=\sum_{m\neq 0}\frac{V_{m0}}{E_0^{(0)}-E_m^{(0)}}\psi_m^{(0)}=\sum_{m\neq 0}a_{0m}\psi_m^{(0)}$$

$$a_{0m}=\frac{V_{m0}}{E_0^{(0)}-E_m^{(0)}}=\frac{\int\psi_m^{(0)}(-q\varepsilon x)\psi_0^{(0)}\,dx}{-\frac{\hbar^2}{2I}m^2}$$

$$=\frac{\left(\frac{1}{2\pi}\right)(-q\varepsilon)}{-\frac{\hbar^2}{2I}m^2}\int_0^{2\pi}e^{-im\phi}\,xdx$$

$m=1$，$a_{01}=\dfrac{Iq\varepsilon R}{\hbar^2}$

$m=-1$，$a_{0-1}=\dfrac{Iq\varepsilon R}{\hbar^2}$

$$\therefore\psi_0^{(1)}(\phi)=a_{01}\psi_1^{(0)}+a_{0-1}\psi_{-1}^{(0)}=\frac{Iq\varepsilon R}{\hbar^2}(\psi_1^{(0)}+\psi_{-1}^{(0)})$$

因此

$$\psi_0'=\psi_0^{(0)}+\psi_0^{(1)}=\psi_0^{(0)}+\frac{Iq\varepsilon R}{\hbar^2}(\psi_1^{(0)}+\psi_{-1}^{(0)})$$

6.

(a) 未微擾位勢的薛丁格方程式為

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2}=E\psi$$

方程式解為

$\psi_n(x)=\sqrt{\dfrac{2}{a}}\cos\left(\dfrac{n\pi}{a}x\right)$，$n=1, 3, 5\cdots$

$\psi_n(x)=\sqrt{\dfrac{2}{a}}\sin\left(\dfrac{n\pi}{a}x\right)$，$n=2, 4, 6\cdots$

$E_n=\dfrac{\pi^2\hbar^2}{2ma^2}n^2$，$n=1, 2, 3\cdots$

$\therefore E_1^{(0)}=\dfrac{\pi^2\hbar^2}{2ma^2}$，$\psi_1^{(0)}=\sqrt{\dfrac{2}{a}}\cos\left(\dfrac{\pi}{a}x\right)$

(b) $E_1^{(1)}=\langle\,\psi_1^{(0)}|V_0|\psi_1^{(0)}\,\rangle$

$$= V_0\left(\frac{2}{a}\right)\int_{-a/4}^{a/4}\cos\left(\frac{\pi}{a}x\right)dx = \frac{\pi^2\hbar^2}{4ma^2}\left[\frac{1}{4}+\frac{1}{2\pi}\right]$$

$$E_1' = E_1^{(0)} + E_1^{(1)} = \frac{\pi^2\hbar^2}{2ma^2}\left(\frac{9}{8}+\frac{1}{4\pi}\right)$$

7.

未微擾位勢時的特徵函數與特徵能量為

$$\psi_n^{(0)}(x) = \sqrt{\frac{2}{b}}\sin\left(\frac{n\pi}{b}x\right)，n = 0, 1, 2, 3\cdots$$

$$E_n^{(0)} = \frac{\pi^2\hbar^2}{2mb^2}n^2，n = 0, 1, 2, 3\cdots$$

所有激發態的能量偏移

$$E_n^{(1)} = \langle\, \psi_n^{(0)}|V(x)|\psi_n^{(0)}\,\rangle$$

$$n \neq 0 \quad E_n^{(1)} = \frac{2\varepsilon}{b}\int_0^b \sin\left(\frac{\pi x}{b}\right)\sin^2\left(\frac{n\pi}{b}x\right)dx$$

$$= \frac{2\varepsilon}{b}\frac{4n^2}{4n^2-1}$$

8.

(a) 利用正交歸一條件求出 c

$$\int_{-\alpha}^{+\alpha}\psi_t^*(x)\,\psi_t(x)\,dx = c^2\int_{-\alpha}^{+\alpha}(\alpha-|x|)dx = c^2\alpha^2 = 1$$

$$\therefore c = \frac{1}{\alpha}$$

$$\psi_t(x) = \frac{1}{\alpha}(\alpha-|x|)$$

(b) $$\langle\, H\,\rangle = \int_{-\alpha}^{\alpha}\psi_t^* g|x|\,\psi_t(x)\,dx = \frac{g}{\alpha^2}\int_{-\alpha}^{\alpha}|x|\,(\alpha-|x|)dx$$

$$= 2\frac{g}{\alpha^2}\int_0^{\alpha}x\,(\alpha-x)dx = \frac{1}{3}g\alpha$$

$$\frac{\partial\,\langle\, H\,\rangle}{\partial\alpha} = \frac{1}{3}g \neq 0$$

因此應返回原先的基本定義，為

$$\langle H \rangle = \int \left[\frac{\hbar^2}{2m} \nabla \psi^* \cdot \nabla \psi + V\psi^* \psi \right] dx$$

$$= \int_{-\alpha}^{\alpha} \left[\frac{\hbar^2}{2m} \left(\frac{1}{\alpha^2} \right) + g|x| \frac{1}{\alpha^2} (\alpha - |x|)^2 \right] dx$$

$$= \frac{\hbar^2}{2m} \frac{(2\alpha)}{\alpha^2} + \frac{2g}{\alpha^2} \int_0^{\alpha} x (x - \alpha)^2 \, dx$$

$$= \frac{\hbar^2}{2m} + \frac{1}{6} g\alpha^2$$

$$\frac{\partial \langle H \rangle}{\partial \alpha} = -\frac{\hbar^2}{m\alpha^2} + \frac{1}{3} g\alpha = 0 \text{，} \alpha^3 = \left(\frac{3\hbar^2}{mg} \right)$$

$$\therefore E_{gs} = \langle H \rangle = \frac{3\hbar^2}{2m} \left(\frac{mg}{3\hbar^2} \right)^{1/3} = \left(\frac{9}{8} \right)^{1/3} \left(\frac{\hbar^4 g}{m^2} \right)^{1/3}$$

9.

$$\langle H \rangle = \int_{-\infty}^{\infty} \psi_t^* \left[\frac{p^2}{2m} + \lambda x^4 \right] \psi_t \, dx$$

$$= \int_{-\infty}^{\infty} \psi_t^* \left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \lambda x^4 \right] \psi_t \, dx$$

$$= -\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \psi_t^* \frac{d^2 \psi_t}{dx^2} \, dx + \lambda \int_{-\infty}^{\infty} \psi_t^* x^4 \psi_t \, dx$$

$$= \frac{\hbar^2}{2m} \sqrt{\frac{\pi}{2}} \alpha^{\frac{1}{2}} + \frac{\lambda}{16} \sqrt{\frac{\pi}{\alpha}} \alpha^{-\frac{5}{2}}$$

$$\frac{\partial \langle H \rangle}{\partial \alpha} = 0 \quad \Rightarrow \quad \alpha^3 = \left(\frac{5\lambda}{16} \right) \left(\frac{2m}{\hbar^2} \right) \text{，} \alpha = \left(\frac{5\lambda}{16} \right)^{1/3} \left(\frac{2m}{\hbar^2} \right)^{1/3}$$

因此

$$E_{gs} = \langle H \rangle \Big|_{\alpha} = \sqrt{\frac{\pi}{2}} \frac{6}{5} \left(\frac{5\lambda}{16} \right)^{\frac{1}{6}} \left(\frac{\hbar^2}{2m} \right)^{5/6} > E_0$$